SYMPLECTIC SINGULARITIES IN GEOMETRY AND REPRESENTATION THEORY, 4-8 APRIL, 2022
CIRM, (LUMINY, MARSEILLE)

	Lundi	Mardi	Mercredi	Jeudi	Vendredi
9h30-10h30	AC	IG	IG	AC	IG
11h00-12h00	Camere	AC	Gammelgaard	IG	Dudas
15h30-16h30	AC	Exercises AC		Exercises IG	
17h00-18h00	Grossi	Mauri		Bertini	
18h15-19h15	Norton	Boissière		Thiel	

AC=Alastair Craw, IG=Iain Gordon.

Titles and Abstracts

Valeria Bertini (TU Chemnitz, Germany),
Hodge structure of O'Grady's singular moduli spaces. Starting from a question of Bakker and C. Lehn about the singularities type of O'Grady's singular moduli spaces, we investigated their Hodge structure. As consequence of our (partial) computation of it, we deduced that O'Grady's moduli spaces do not have finite quotient singularities. In this talk I will present the techniques we used for the computation of their Hodge structure, and then of some of their Betti numbers and their Euler characteristic, and I will focus on the relevant properties giving consequences on their singularities type.

This is a joint work with Franco Giovenzana.

Samuel Boissière (University of Poitiers, France),
The singularities of the Fano variety of lines of a nodal cubic fourfold. In the context of the compactifications of the moduli spaces of prime order non-symplectic automorphisms of hyperkaehler manifolds, a key question is to understand the correct notion of limit automorphisms. To give an intuition of the geometry involved in this research, I will present some classical results involving K3 surfaces defined as a double cover of the plane branched over a sextic that degenerates to a nodal one. Then I will show a similar property for Fano varieties of lines of singular cubic fourfolds and I will show how the study of the singularities of the Fano variety of lines helps to interpret geometrically the limit automorphism.

This is a joint work in progress with Alessandra Sarti.

Chiara Camere (University of Milan, Italy),
Projective models of Nikulin orbifolds. In this talk, I will first describe families of projective fourfolds of $K3^{[2]}$-type carrying a symplectic involution. To each of these families one can associate a family of projective irreducible symplectic orbifolds, obtained as partial resolution of their quotients, which we call Nikulin orbifolds. I will explain how to study projective models of these orbifolds and finally, if time allows, I will describe a locally complete projective family containing Nikulin orbifolds of degree two.

This is joint work with A. Garbagnati, G. Kapustka and M. Kapustka.

Alastair Craw (University of Bath, UK),
The McKay correspondence and birational geometry. The classical *McKay correspondence* centres on the study of the simplest class of symplectic quotient singularities, namely the Kleinian (or ADE surface) singularities, from both the algebro-geometric and representation-theoretic points of view. The key idea is to construct the unique minimal resolution of the Kleinian singularity by variation of GIT quotient for a class of moduli spaces arising from the McKay quiver. The connection between the representation theory on one hand, and the algebraic geometry on the other, is provided by the *linearisation map* that identifies a Weyl chamber of type ADE with the ample cone of the resolution. A similar pattern emerges in higher dimensions for the symmetric products of Kleinian singularities, also known as wreath product singularities; again, the linearisation map is an isomorphism, but here the birational geometry of a crepant resolution is much more interesting. These examples provide a wonderful playground in which to explore some quite general phenomena, including the Namikawa-Weyl group action, the movable cone of the quiver moduli space, and Mukai flops, together with the fundamental fact that projective crepant resolutions of conical symplectic singularities are *Mori Dream Spaces*.

Olivier Dudas (University of Paris, France),
Nabla operators for complex reflection groups. In the 90's, Bergeron and Garsia introduced a remarkable operator on the ring of symmetric functions. Lots of interesting combinatorics are attached to that operator. For example, it can be used to define a bigraded version of the Catalan numbers, or to compute the bigraded character of the diagonal coinvariants. The work of Haiman around the n!-conjecture provides a geometric interpretation of that operator. It is closely related to the "nice" properties of the symplectic resolution of C^{2n}/S_n.

In this talk I will explain how one can hope to extend that description when S_n is any other finite complex reflection group, even though no symplectic resolution exists. If time permits, I will draw some consequences for the representation theory of finite reductive groups.

This is a work in progress with R. Rouquier and C. Stump.

Soren Gammelgaard (University of Oxford, UK),
Quiver varieties and moduli spaces attached to Kleinian singularities. We discuss how Nakajima's quiver varieties can be used to understand the Hilbert schemes of points on Kleinian singularities, that is, singular surfaces isomorphic to C^2/G, for G a finite subgroup of $SL_2(C)$.

This approach lets us show that each such Hilbert scheme is irreducible, has symplectic singularities, and has a unique symplectic resolution. Time permitting, we may touch upon a type of "equivariant Quot scheme" associated to the same singularity, or possibly a generalisation to sheaves on a stacky compactification of the same singular surface.

This is joint work with A. Craw, Á. Gyenge, and B. Szendröi.

Iain Gordon (University of Edinburgh, UK),
Symplectic representation theory. Symplectic quotient singularities and their resolutions, which will be discussed in Alastair Craw's talks, have algebras attached to them by quantization. The representation theory of these algebras is interesting, for their own sake and also for feeding back some information to algebraic geometry as well as providing new ways to look at other parts of geometric representation theory. These talks will introduce the types of quantization studied and discuss some of the associated representation theory. This includes examples around wreath product singularities as in Alastair's talks (Cherednik

algebras), category O. This approach leads also to a surprising symplectic duality and new families of *Coulomb branch varieties.*

Annalisa Grossi (TU Chemnitz, Germany),
Ihs varieties as symplectic quotients of ihs manifolds. We aim at constructing ihs varieties with trivial algebraic regular fundamental group starting from an ihs manifold X and a finite group G of symplectic actions on X. I will present some work in progress in the case where X is a generalized Kummer fourfold or an O'Grady's sixfold. The case of generalized Kummer fourfolds is the content of a joint work with Bertini, Capasso, Mauri and Mazzon. Moreover I will present some classification results about symplectic birational transformations of manifolds of OG_6–type (joint work with Onorati and Veniani) and how these results could be exploit for our purpose.

Mirko Mauri (University of Michigan, USA),
Hodge-to-singular correspondence. The decomposition theorem for proper morphisms of algebraic varieties grants that the cohomology of the domain splits in elementary summands. However, in general, it is a subtle task to determine explicitly these summands. We prove that this is in fact possible in the case of Hitchin fibrations for Higgs bundles of arbitrary degree on the locus of reduced spectral curves. Surprisingly we relate the summands of the decomposition theorem to the topology of symplectic singularity on the moduli spaces of Higgs bundles in (fixed!) degree zero.

This is based on a collaboration with Luca Migliorini.

Emily Norton (University of Kent, UK),
Characters of rational Cherednik algebra modules and applications. Rational Cherednik algebras associated to complex reflection groups W are the most studied type of symplectic reflection algebras, defined by Etingof and Ginzburg. They have a category O of representations which contains all finite-dimensional representations. The finite-dimensional representations show up in knot theory, cohomology of affine Springer fibers, and diagonal coinvariant rings. In this talk I will discuss, for W a Coxeter group of exceptional type, how to find the characters of these representations (old solo work) and potential applications to understanding the ring of diagonal coinvariants of type W (work in progress with Stephen Griffeth).

Ulrich Thiel (University of Kaiserslautern, Germany),
Computational aspects of Calogero-Moser spaces. I will do a show & tell on algorithmic approaches to some of the topics covered in the courses by Alastair Craw and Iain Gordon. On the one hand, I hope this makes things more concrete and accessible. On the other hand, I hope this convinces you that the computer can be a helpful tool in your research as well.