ZELEVINSKY INVOLUTION AND MOEGLIN-WALDSPURGER ALGORITHM FOR $\mathrm{GL}_n(D)$

A. I. BADULESCU AND D. RENARD

Abstract. In this short note, we remark that the algorithm of Moeglin and Waldspurger for computing the dual (as defined by Zelevinsky) of an irreducible representation of GL_n still works for the inner forms of GL_n, the proof being basically the same.

1. Segments, multisegments and the involution

A **multiset** is a finite set with finite repetitions $(a, a, b, c, d, d, d, e, a, ...)$. A **segment** Δ is the void set or a set of consecutive integers $\{b, b+1, ..., e\}$, $b, e \in \mathbb{Z}$, $b \leq e$. We call e the **ending** of Δ and the integer $e - b + 1$ the **length** of Δ. By convention, the length of the void segment is 0. Let $\Delta = \{b, b+1, ..., e\}$ and $\Delta' = \{b', b'+1, ..., e'\}$ be two segments. We say Δ **precede** Δ' if $b < b'$, $e < e'$ and $b' \leq e+1$. We also write $\Delta \geq \Delta'$ if $b > b'$ or $b = b'$ and $e \geq e'$. This is a total order on the set of segments.

A **multisegment** is a multiset of segments. We identify multisegments obtained from each other by dropping or adding void segments. The **full extended length** of a multisegment is the sum of the lengths of all its elements and is 0 if the multisegment is void. The **support** of a multisegment m is the multiset of integers obtained by taking the union (with repetitions) of the segments in m. A multisegment $(\Delta_1, \Delta_2, ..., \Delta_k)$ is said to be ordered if $(\Delta_1 \geq \Delta_2 \geq ... \geq \Delta_t)$. The lexicographic order induces a total order on ordered multisegments : if $m = (\Delta_1, \Delta_2, ..., \Delta_t)$ and $m' = (\Delta'_1, \Delta'_2, ..., \Delta'_{t'})$ are multisegments, then $m \geq m'$ if $\Delta_1 > \Delta'_1$, or $\Delta_1 = \Delta'_1$ and $\Delta_2 > \Delta'_2$, and so on, or $t \geq t'$ and $\Delta_i = \Delta'_i$ for all $i \in \{1, 2, ..., t'\}$.

If $\Delta = \{b, b+1, ..., e\}$ is a segment, we set $\Delta^- = \{b, b+1, ..., e-1\}$ with the convention that Δ^- is void if $b = e$.

Let m be a multisegment. We associate to m a multisegment $m^\#$ in the following way : let d be the biggest ending of a segment in m. Then chose a segment Δ_{i_0} in m containing d and maximal for this property. Then we define the integers $i_1, i_2, ..., i_r$ inductively : Δ_{i_s} is a segment of m preceding $\Delta_{i_{s-1}}$ with ending $d - s$, maximal with these properties, and r is such that there's no possibility to find such a i_{r+1}. Set $m^- = (\Delta'_1, \Delta'_2, ..., \Delta'_t)$, where

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$\Delta'_i = \Delta_i$ if $i \notin \{i_0, i_1, ..., i_r\}$, and $\Delta'_i = \Delta_i^-$ if $i \notin \{i_0, i_1, ..., i_r\}$. Then $\{d-r, d-r+1, ..., d\}$ is the first segment of $m^\#$. Starting from the beginning with m^- what we have done with m, we find the second segment of $m^\#$, and so on (so at the end we have that $m^\#$ is the multiset union of $\{d-s, d-s+1, ..., d\}$ and $(m^-)^\#$). This multisegment $m^\#$ is independent of the choices made for the construction. The map $m \mapsto m^\#$ is an involution of the set of non void multisegments. It preserves the support.

2. Representations of G_n

2.1. **Generalities.** Let F be an non-Archimedean local field of any characteristic with norm $|\ |_F$. For all $n \in \mathbb{N}^*$ let G_n be the group $GL_n(F)$, $\mathcal{A}_n$ be the set of equivalence classes of smooth finite length representations of G_n and $\mathcal{R}_n$ be the Grothendieck group of smooth finite length representations of G_n. As usual, we will slightly abuse notation by identifying representations and their equivalence classes, and sometimes, representations with their image in the Grothendieck group $\mathcal{R}_n$.

The set B_n of classes of smooth irreducible representations of G_n is a basis of $\mathcal{R}_n$. If $\pi_1 \in B_{n_1}$ and $\pi_2 \in B_{n_2}$, then $\pi_1 \otimes \pi_2$ is a representation of $G_{n_1} \times G_{n_2}$. This group may be seen as the subgroup L of matrices diagonal by two blocks of size n_1 and n_2 of $G_{n_1+n_2}$. We set

$$\pi_1 \times \pi_2 = \mathrm{ind}_P^{G_{n_1+n_2}}(\pi_1 \otimes \pi_2)$$

where "ind" is the normalized parabolic induction functor and P is the parabolic subgroup of $G_{n_1+n_2}$ containing L and the group of upper triangular matrices. We generalize this notation in an obvious way to any finite number of elements $\pi_i \in B_{n_i}$, $i \in \{1, 2, ..., k\}$.

Let $\mathcal{C}_n$ be the set of cuspidal representations of G_n and $\mathcal{D}_n$ the set of essentially square integrable representations of G_n (we assume irreducibility in the definition of cuspidal and essentially square integrable representations).

If χ is a smooth character of G_n and $\pi \in \mathcal{A}_n$, then $\chi\pi$ will denote the tensor product representation $\chi \otimes \pi$. Let ν_n be the character $g \mapsto |det(g)|_F$ of G_n. We will drop the index n when no confusion may occur.

2.2. **Irreducible representations.** Let $k \in \mathbb{N}^*$ and n_i, $i \in \{1, 2, ..., k\}$ be positive integers. For each i let $\sigma_i \in \mathcal{D}_{n_i}$. The representations σ_i being essential square integrable, for all $i \in \{1, 2, ..., k\}$ there exists a unique real number a_i such that $\nu^{a_i}\sigma_i$ is unitary. If the σ_i are ordered such that the sequence a_i is increasing, then $S = \sigma_1 \times \sigma_2 \times ... \times \sigma_k$ is called a **standard representation** and has a unique irreducible quotient $\theta(S)$. The representation S doesn't depend on the order of the σ_i as long as the condition that the sequence a_i is increasing is fullfiled. So S and $\theta(S)$ depend only on the multiset $(\sigma_1, \sigma_2, ..., \sigma_k)$. We call this multiset the **esi-support** of S or of $\theta(S)$ ("esi" : essentially square integrable).

2.3. **Standard elements.** The image in $\mathcal{R}_n$ of a standard representation is called a **standard element** of $\mathcal{R}_n$. The set H_n of standard elements of

$\mathcal{R}_n$ is a basis of $\mathcal{R}_n$. The map $W_n : S \mapsto \theta(S)$ is a bijection from H_n to B_n (see [DKV]).

2.4. **The involution.** On $\mathcal{R}_n$, we consider the involution I_n from [Au], which transforms irreducible representations to irreducible representations up to a sign. The involution commutes with induction ([Au]), i.e. if $\pi_1 \in B_{n_1}$ and $\pi_2 \in B_{n_2}$, then $I_{n_1+n_2}(\pi_1 \times \pi_2) = I_{n_1}(\pi_1) \times I_{n_2}(\pi_2)$. Forgetting signs, the involution in [Au] gives rise to a permutation $|I_n|$ of B_n (which is the involution defined in [Ze]). We will call $|I_n|(\pi)$ the **dual** of π. See [Au] and [Ze].

The algorithm of Moeglin and Waldspurger ([MW]) computes the esi-support of the dual of a smooth irreducible representation π from the esi-support of π.

2.5. **Essentially square integrable representations.** Following [Ze], if k is a positive integer such that $k|n$, if we set $p = n/k$ and chose $\rho \in \mathcal{C}_p$, then $\rho \times \nu\rho \times \nu^2\rho\times, ..., \times\nu^{k-1}\rho$ has a unique irreducible quotient $Z(k, \rho)$ which is an essentially square integrable representation of G_n. Any element σ of $\mathcal{D}_n$ is obtained in this way and σ determines k and ρ such that $\sigma = Z(k, \rho)$. If $\rho \in \mathcal{C}_p$ for some p, given a segment $\Delta = \{b, b + 1, ..., e\}$, we set

$$< \Delta >_\rho = Z(\nu^b\rho, e - b + 1) \in \mathcal{D}_{p(e-b+1)}.$$

2.6. **Rigid representations.** If $\rho \in \mathcal{C}_p$ for some p we call the set $\{\nu^k\rho\}_{k\in\mathbb{Z}}$ **the ρ-line**. If $\pi \in B_n$ we say π is **ρ-rigid** if the cuspidal support of π is included in the ρ-line (of course, it is the $\nu\rho$-line too). An irreducible representation is called **rigid** if it is ρ-rigid for some ρ. If $\pi_1 \in B_{n_1}$ and $\pi_2 \in B_{n_2}$ are such that the cuspidal supports of π_1 and π_2 are disjoint, then $\pi_1 \times \pi_2$ is irreducible. So any $\pi \in B_n$ is a product of rigid representations π_i. Then we know ([Ze]) that the esi-support of π is the reunion with multiplicities of the esi-supports of the π_i. As I_n commutes with induction, to compute the esi-support of duals of irreducible representations, we need only to compute the esi-support of duals of rigid representations.

2.7. **Multisegments and representations.** If $m = (\Delta_1, \Delta_2, ..., \Delta_k)$ is an ordered multisegment of full length q and $\rho \in \mathcal{C}_p$, then m and ρ define a standard element $\pi_\rho(m)$ of $\mathcal{R}_{pq}$, precisely

$$\pi_\rho(m) =< \Delta_1 >_\rho \times < \Delta_2 >_\rho \times, ..., \times < \Delta_k >_\rho \in H_{pq},$$

and an irreducible representation

$$< m >_\rho = W_n(\pi_\rho(m)) \in B_{pq}.$$

The map $m \mapsto < m >_\rho$ realizes a bijection between the set of multisegments of full length q and the set $B_{n,\rho}$ of ρ-rigid irreducible representations of G_{pq}.

2.8. **The algorithm for G_n.** The result of Moeglin and Waldspurger in [MW] is : the dual of $< m >_\rho$ is $< m^\# >_\rho$.

2.9. **The proof.** We recall here their argument:

Let (p,ρ) be a couple such that p is a positive integer and $\rho \in \mathcal{C}_p$. Fix a multiset s with integer entries, and let S be the (finite) set of all the multisegments m having support s. They all have the same full length, let's call it k. Set $n = pk$. Let $B_\rho = \{< m >_\rho, m \in S\}$ and $H_\rho = \{\pi_\rho(m), m \in S\}$. Let $\mathcal{R}_\rho$ be the (finite dimensional) submodule of $\mathcal{R}_n$ generated by B_ρ. Then B_ρ and H_ρ are basis of the space $\mathcal{R}_\rho$. On B_ρ and H_ρ consider the decreasing order induced by the order on multisegments in S. Then we know that for this order the matrix M of H_ρ in the basis B_ρ is upper triangular and unipotent ([Ze] or [DKV]). The space $\mathcal{R}_\rho$ is stable under I_n. It is important to notice here that the involution $(-1)^{n-k}I_n$ of $\mathcal{R}_\rho$ transforms every irreducible representation in an irreducible one, since all the elements here have the same cuspidal support, of full length k (see [Au]). In other words, the restriction of $|I_n|$ to B_ρ is $(-1)^{n-k}I_n$.

Let T_1 (resp. T_2) be the matrix of the involution $(-1)^{n-k}I_n$ of $\mathcal{R}_\rho$ in the basis B_ρ (resp. H_ρ). *Then the matrix T_1 doesn't depend on the couple (p,ρ).* The argument, attributed in [MW] to Oesterlé, is the following:

We have already seen that T_1 is a permutation matrix ([Au]). Then as M is an upper triangular unipotent matrix, the relation $T_2 = M^{-1}T_1M$ is a Bruhat decomposition for T_2 and this implies that T_1 is determined by T_2.

Now, T_2 itself doesn't depend on the couple (p,ρ) because:

(c1) if $m = (\Delta_1, \Delta_2, ..., \Delta_t)$ with Δ_i of length n_i/p, then

$$I_n(\pi_\rho(m)) = I_{n_1}(< \Delta_1 >_\rho) \times I_{n_2}(< \Delta_2 >_\rho) \times ... \times I_{n_t}(< \Delta_t >_\rho),$$

(c2) if $\Delta = \{b, b+1, ..., e\}$, then

$$I_{(e+1-b)p}(< \Delta >_\rho) = (-1)^{(e+1-b)(p-1)} < m_\Delta >_\rho,$$

where $m_\Delta = (\{b\}, \{b+1\}, ..., \{e\})$,

(c3) one has $< m_\Delta >_\rho = \sum_{m' \leq m_\Delta} (-1)^{d(m')+e-b+1}\pi_\rho(m')$, where $d(m')$ is the cardinality of m' (as a multiset of segments) ([Ze]).

So it is enough to show that the dual of $< m >_\rho$ is $< m^\# >_\rho$ for a particular ρ. The authors conclude their proof by showing this relation holds for a clever choice of the cuspidal representation ρ.

3. Representations of G'_n

Let D be a central division algebra of dimension d^2 over F (with $d \in \mathbb{N}^*$) and let G'_n be the group $GL_n(D)$. We use the notation for objects relative to G_n, but with a prime, for objects relative to G'_n : $\mathcal{A}'_n$, $\mathcal{C}'_n$, $\mathcal{D}'_n$, $\mathcal{R}'_n$, B'_n... The involution I'_n ([Au]) on $\mathcal{R}'_n$, has the same properties as I_n : it transforms irreducible representations into irreducible representations, up to a sign, and commutes with induction.

If $g' \in G'_n$, one can define the characteristic polynomial $P_{g'} \in F[X]$ of g', and $P_{g'}$ is monic of degree nd ([Pi]). If $g' \in G'_n$, the determinant $det(g')$ of g' is the constant term of its characteristic polynomial. We write ν'_n for the character $g' \mapsto |det(g')|_F$ of G'_n, and we drop the index n when no confusion may occur.

For a given n, if $g \in G_{nd}$ and $g' \in G'_n$ we write $g \leftrightarrow g'$ if the characteristic polynomial of g is separable (i.e. has distinct roots in an algebraic closure of F) and is equal to the characteristic polynomial of g'. If $\pi \in \mathcal{R}_{nd}$ or $\pi \in \mathcal{R}'_n$, we denote by χ_π the character of π. It is well defined on the set of elements with separable characteristic polynomial even if the characteristic of F is not zero. The Jacquet-Langlands correspondence is the following result :

Theorem 3.1. *There exists a unique bijection* $\mathbf{C} : \mathcal{D}_{nd} \to \mathcal{D}'_n$ *such that for all* $\pi \in \mathcal{D}_{nd}$ *one has*

$$\chi_\pi(g) = (-1)^{nd-n} \chi_{\mathbf{C}(\pi)}(g')$$

for all $g \leftrightarrow g'$.

This well known result of [DKV] is also true in non-zero characteristic ([Ba1]).

One can extend the Jacquet-Langlands correspondence to a linear map between Grothendieck groups ([Ba2]) :

Proposition 3.2. *a) There exists a unique group morphism* $\mathbf{LJ} : \mathcal{R}_{nd} \to \mathcal{R}'_n$ *such that for all* $\pi \in \mathcal{R}_{nd}$ *one has*

$$\chi_\pi(g) = (-1)^{nd-n} \chi_{\mathbf{LJ}(\pi)}(g')$$

for all $g \leftrightarrow g'$.

The morphism $\mathbf{LJ}$ *is defined on the basis* H_{nd} *: if* $S = \sigma_1 \times \sigma_2 \times ... \times \sigma_k$, *with* $\sigma_i \in \mathcal{D}_{n_i}$, *then*

- if for all $i \in \{1, 2, ..., k\}$, $d | n_i$,

$$\mathbf{LJ}(S) = \mathbf{C}(\sigma_1) \times \mathbf{C}(\sigma_2) \times ... \times \mathbf{C}(\sigma_k),$$

- if not, $\mathbf{LJ}(S) = 0$.

b) For all $\pi \in \mathcal{R}_{nd}$, $\mathbf{LJ}(I_{nd}(\pi)) = (-1)^{nd-n} I'_n(\mathbf{LJ}(\pi))$.

The classification of irreducible representations is similar to the one for G_n, and we can define the esi-support of an irreducible representation, the standard elements H'_n and the bijection $W'_n : H'_n \to B'_n$. Knowing the esi-support of $\pi' \in B'_n$, one would like to compute the esi-support of $|I'_n|(\pi')$.

The classification of essentially square integrable representations on G'_n differs slightly from that on G_n (it is more general, since $G'_n = G_n$ when $D = F$). If $\rho' \in \mathcal{D}'_n$, then $\mathbf{C}^{-1}(\rho') \in \mathcal{D}_{nd}$. Following [Ta], if $\mathbf{C}^{-1}(\rho') = Z(k, \rho)$, we set $s(\rho') = k$, and $\nu_{\rho'} = (\nu')^{s(\rho')}$. Given a positive integer k such that $k|n$ and a $\rho' \in \mathcal{C}'_p$ where $p = n/k$, the representation $\rho' \times \nu_{\rho'}\rho \times \nu^2_{\rho'}\rho' \times ... \times \nu^{k-1}_{\rho'}\rho'$ has a unique irreducible quotient σ' which is an essentially square integrable representation of G'_n. We set then $\sigma' = T(k, \rho')$. Any $\sigma' \in \mathcal{D}'_n$ is obtained in this way and σ' determines k and ρ' such that $\sigma' = T(k, \rho')$. See [Ta] for details.

If $\rho' \in \mathcal{C}_p$ for some p, given a segment $\Delta = \{b, b+1, ..., e\}$, we set

$$< \Delta >_{\rho'} = T(\nu^b_{\rho'}\rho', e - b + 1) \in \mathcal{D}'_{p(e-b+1)}.$$

A line in this setting is a set of the form $\{\nu^k_{\rho'}\rho'\}_{k \in \mathbb{Z}}$ where ρ' is a cuspidal representation. The definition of ρ'-rigid and rigid representations and their properties are similar to the ones for G_n, and as for G_n, one needs only to compute of the esi-support of the duals for rigid representations.

If $m = (\Delta_1, \Delta_2, ..., \Delta_k)$ is an ordered multisegment and $\rho' \in \mathcal{C}'_p$, then m and ρ' define a standard element of some $\mathcal{R}'_n$, more precisely

$$\pi'_{\rho'}(m) =< \Delta_1 >_{\rho'} \times < \Delta_2 >_{\rho'} \times, ..., \times < \Delta_k >_{\rho'},$$

and an irreducible representation

$$< m >_{\rho'}= W'_n(\pi'_{\rho'}(m)).$$

The map $\pi'_{\rho'}$ realizes a bijection between the set of multisegments of full length k and the set of ρ'-rigid representations of G'_{pk}. Now, we claim that the algorithm for G'_n is the same as for G_n, namely :

Theorem 3.3. *The dual of the representation $< m >_{\rho'}$ is $< m^\# >_{\rho'}$.*

For the proof, we follow the argument in [MW] :

Let (p, ρ') be a couple such that p is a positive integer and $\rho \in \mathcal{C}'_p$, let k be a positive integer and set $n = pk$. Let $B'_{\rho'} = \{< m >_{\rho'}, m \in S\}$ and $H'_{\rho'} = \{\pi'_{\rho'}(m), m \in S\}$ (S has already been defined in the section 2.9). Let $\mathcal{R}'_{\rho'}$ be the finite dimensional submodule of $\mathcal{R}'_n$ generated by $B'_{\rho'}$. Then $B'_{\rho'}$ and $H'_{\rho'}$ are bases of $\mathcal{R}'_{\rho'}$. On $B'_{\rho'}$ and $H'_{\rho'}$ consider the decreasing order induced by the order on multisegments in S. Then the matrix M' of $H'_{\rho'}$ in the basis $B'_{\rho'}$ is upper triangular and unipotent ([DKV] and [Ta]). The involution $(-1)^{n-k} I'_n$ induces an involution of $\mathcal{R}'_{\rho'}$ which carries irreducible representations to irreducible representations. Let T'_1 (resp. T'_2) be the matrix of this involution in the basis $B'_{\rho'}$ (resp. $H'_{\rho'}$).

As for G_n, the matrix T'_1 doesn't depend on (p, ρ'), because Oesterlé's argument works again. First of all (see [Au]), T'_1 is a permutation matrix so the relation $T'_2 = M'^{-1} T'_1 M'$ is a Bruhat decomposition for T'_2 and this implies that T'_1 is determined by T'_2.

As for G_n, T'_2 itself doesn't depend on (p, ρ') because, as we will explain shortly afterwards, we have :

(c'1) If $m = (\Delta_1, \Delta_2, ..., \Delta_t)$ with Δ_i of length n_i/p, then

$$I'_n(\pi'_{\rho'}(m)) = I'_{n_1}(< \Delta_1 >_{\rho'}) \times I'_{n_2}(< \Delta_2 >_{\rho'}) \times ... \times I'_{n_t}(< \Delta_t >_{\rho'}).$$

(c'2) If $\Delta = \{b, b+1, ..., e\}$, then

$$I'_{(e+1-b)p}(< \Delta >_{\rho'}) = (-1)^{(e+1-b)(p-1)} < m_\Delta >_{\rho'},$$

where $m_\Delta = (\{b\}, \{b+1\}, ..., \{e\})$.

(c'3) One has $< m_\Delta >_{\rho'}= \sum_{m' \leq m_\Delta} (-1)^{d(m')+e-b+1} \pi'_{\rho'}(m')$, where $d(m')$ is the cardinality of m' (as a multiset of segments).

The relation (c'1) is clear since the involution commutes with induction ([Au]).

(c'2) is true too: from the formula for I'_n to be found in [Au], and the computation in [DKV] of all normalized parabolic restrictions of essentially square integrable representations of G'_n, one may see $I'_{(e+1-b)p}(< \Delta >_{\rho'})$ is an alternate sum of representations $\pi'_{\rho'}(m_i)$, where m_i runs over the set of multisegments with same support as Δ. It is obvious that the maximal one is $\pi'_{\rho'}(m_\Delta)$. It appears in the sum with coefficient $(-1)^{(e+1-b)(p-1)}$, and so $W'_n(\pi'_{\rho'}(m_\Delta)) =< m_\Delta >_{\rho'}$, has to appear with coefficient $(-1)^{(e+1-b)(p-1)}$ in the final result. As we know a priori that this result is plus or minus an irreducible representation, (c'2) follows.

(c'3) is the combinatorial inversion formula ([Ze]), which is still true here since for all $m' \leq m_\Delta$ one has $\pi'_{\rho'}(m') = \sum_{m'' \leq m'} < m'' >_{\rho'}$.

So it is enough to show that the dual of $< m >_{\rho'}$ is $< m^{\#} >_{\rho'}$ for a particular ρ'. Or, equivalently, to show that for some ρ' we have $T'_2 = T_2$. Let $\rho \in \mathcal{C}_d$ and set $\rho' = \mathbf{C}(\rho)$. Then $\rho' \in \mathcal{C}'_1$ and $s(\rho') = 1$. The map **LJ** induces a bijection from H_ρ to $H'_{\rho'}$ commuting with the bijections from S onto these sets. The point b) of the proposition 3.2 implies then $T'_2 = T_2$.

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Département de Mathématiques, Université de Poitiers, Téléport 2, Boulevard Marie et Pierre Curie, BP 30179, 86962 Futuroscope Cedex, France

E-mail address: badulesc@math.univ-poitiers.fr

Centre de Mathématiques Laurent Schwartz, Ecole Polytechnique, 91128 Palaiseau Cedex Cedex, France

E-mail address: renard@math.polytechnique.fr