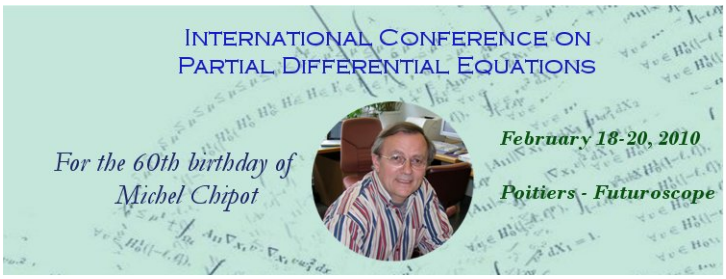

Parallell Sessions

1 Theory, existence, comparison...

Elliptic problems in the half-space

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The aim of this talk is the resolution of some elliptical prolems in the half-space $\mathbb{R}^N_+$, with $N \geq 2$. Using the Dirichlet and Neumann problems for the Laplace operator, we give existence, uniqueness and regularity results in L^p theory for the biharmonic and Stokes problems (see Amrouche-Raudin J. Diff. Eq. 2007, Com. Pure Appl. Math. 2007, Amrouche-Necasova-Raudin J. Diff. Eq. 2008 and SIAM J. Math. Anal. 2010). For that, we consider data and give solutions which live in weighted Sobolev spaces. We assume that the boundary conditions are nonhomogeneous and we also take them in weighted Sobolev spaces. An important aspect of this study is the case of singular boundary conditions and the very weak solutions which correspond to it. We also treat the question of non standard boundary conditions. A part of this work is devoted to the study of the reflection principles for the biharmonic and Stokes operators. We give weak formulations of these principles with the aim of getting the kernels in some distribution spaces. In a second part, we obtain new estimates for L^1 and L^N vector fields, which yield in particular improved estimates for the solution of elliptic systems in the half-space (see Bourgain-Brzis J. AMS 2002, J. European Math. Soc. 2007 and Brzis- Van Schaftingen J. of the Eur. Math. Soc. 2007).

Hyperbolic equation with a memory term nonlinearity

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We study the local existence, using Strichartz's estimates, of the solution of the following hyperbolic equation with a nonlocal in time nonlinearity:

$$u_{tt} - \Delta u = \frac{1}{\Gamma(1-\gamma)} \int_0^t (t-s)^{-\gamma} |u(s)|^p \, ds \quad x \in \mathbb{R}^N, t > 0,$$

$$u(0,x) = u_0(x), \; u_t(0,x) = u_1(x) \qquad\qquad x \in \mathbb{R}^N,$$

where $(u_0, u_1) \in H^\mu(\mathbb{R}^N) \times H^{\mu-1}(\mathbb{R}^N)$, $0 < \mu \leq N/2$, $N \geq 2$, $0 < \gamma < 1$, $p > 1$, $H^\mu(\mathbb{R}^N)$ is the homogeneous Sobolev space of order μ and Γ is the Euler gamma function.
Moreover, we give a non-existence theorem of the global solution in whole space, under some conditions on the initial value without any conditions on the support.

About a coupled elliptic-hyperbolic set pf PDEs for the modelling of bubbles in nuclear reactors

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The system we obtain from an asymptotic expansion with respect to the Mach Number and from other simplifications is a coupling between a Poisson equation and a transport equation. We present here both theoretical and numerical aspects of the study of this model - called Abstract Bubble Vibration Model - including the proof of a short-time existence theorem with a special attention paid to the expression of the existence time. We also get interested in algebraic properties of the solutions in the case of less smooth initial conditions.

A class of nonlinear elliptic equations with integrable data

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We study a class of nonlinear elliptic equations with two lower order terms and with integrable data. The parabolic version is also investigated. The difficulties connected to this problem are due to the integrable data and to the presence of the two lower order terms which produce a lack of coercivity. Under the framework of renormalized solutions we give existence and uniqueness results.

On the Cauchy problem for nonlinear parabolic equations in the hyperbolic space

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We investigate, in the hyperbolic space, the Cauchy problem for a class of nonlinear parabolic equations, where the density is variable and the diffusion is given by the Laplace-Beltrami operator. At first, we address the corresponding problem in the euclidean space, giving criteria involving the density, for well-posedness. Then, since the Laplace-Beltrami operator on the ball model of the hyperbolic space can be regarded as a degenerate elliptic operator with drift on the euclidean ball, inspired by previous results, we consider the Cauchy problem in the hyperbolic space. Hence we provide conditions for uniqueness in dependence of the behaviour of the density. Finally, we discuss support properties of solutions.

The Fujita phenomenon under dynamical boundary conditions in exterior domains

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The Fujita phenomenon for nonlinear parabolic problems in an exterior domain of $\mathbf{R}^N$ under dissipative dynamical boundary conditions is investigated in the superlinear case. As in the case of Dirichlet boundary conditions, it turns out that there exists a critical exponent $p = 1 + 2/N$ such that blow-up of positive solutions always occurs for subcritical exponents, whereas in the supercritical case global existence can occur for small non-negative initial data.

Liouville type results for linear elliptic and parabolic operators

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In this talk I first present some extensions of the classical Liouville theorem for harmonic functions to solutions of linear parabolic operators with periodic coefficients. I further exhibit an explicit counterexample showing that the periodicity assumption cannot be relaxed by the almost periodicity. This counterexample can also be used to show the non-existence of almost periodic principal eigenfunctions.

Long-time behaviour of weak entropy measure-valued solutions to a class of forward-backward parabolic equations

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We consider the one-dimensional Neumann initial-boundary value problem for a class of forward-backward parabolic equations with a non-linear diffusivity described by a cubic-like function. Since the lack of forward parabolicity gives rise to (possibly) ill-posed problems, a widely accepted idea is to introduce some physically sensible regularization which leads to a family of well-posed (approximating) problems. Here we consider a viscous pseudoparabolic regularization, described by an equation of Sobolev type, whose main feature is to take memory effects into account. As proven by Plotnikov, taking the vanishing viscosity limit in the approximating problems allows to define the class of the weak entropy measure-valued solutions to the original unperturbed problem. This terminology is suggested by the fact that such solutions satisfy a suitable class of inequalities, the so-called entropy inequalities. In this general framework, relying on a strong version of the entropy inequalities, our aim is to establish results concerning the long-time behaviour of such solutions.

Existence and regularity of extremal solutions for a mean-curvature equation

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We study a class of mean curvature equations $-\mathcal{M}u = H+\lambda u^p$ where $\mathcal{M}$ denotes the mean curvature operator and for $p \geq 1$. We show that there exists an extremal parameter λ^* such that this equation admits a minimal weak solutions for all $\lambda \in [0, \lambda^*]$, while no weak solutions exists for $\lambda > \lambda^*$ (weak solutions will be defined as critical points of a suitable functional). In the radially symmetric case, we then show that minimal weak solutions are classical solutions for all $\lambda \in [0, \lambda^*]$ and that another branch of classical solutions exists in a neighborhood $(\lambda_* - \eta, \lambda^*)$ of λ^*.

2 Nonlocal interactions, image processing, singular perturbations...

Asymptotic behaviour for a diffusion equation governed by nonlocal interactions

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We consider a diffusion model in which the distribution is dependent on the extent of long-range interactions and address the question of asymptotic behaviour of the population. We prove firstly the existence, uniqueness and L^∞ estimate from L^p estimate of the parabolic equation. Secondly we study the associated stationary equation and generalize a result of Chipot-Lovat determining the number of stationary solutions. We finish our study by giving in some cases the asymptotic behavior of the solution for large time.

Anisotropic singular perturbations method to solve some integro-differential problems

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The anisotropic singular perturbations problems arise when we consider parametrized PDE's $(P_\varepsilon)_{\varepsilon\geq 0}$ and the limit problem P_0 has a lower derivative order in some directions. This technique can be used as a method to show the existence of solutions of some nonlinear problems. As a type model, we consider the following integro-differential problem defined in the unit square $\Omega = (0,1)\times(0,1)$

$$\begin{cases} -\partial^2_{x_2} u_0(x_1,x_2) = a\left(\int_0^1 u_0(x_1,x_2)\,dx_1\right) & \text{in } \Omega, \\ u_0(x_1,\cdot) = 0 & \text{on } \{0,1\}, \end{cases} \tag{1}$$

where a is a continuous function. This type of problems arises in the neutron transport theory. Using the anisotropic singular perturbations method we can show that there exists u_0 solution to (1) as a limit of u_ε solution to the nonlocal elliptic problem

$$\begin{cases} -\varepsilon^2\partial^2_{x_1} u_\varepsilon - \partial^2_{x_2} u_\varepsilon = a\left(\int_0^1 u_0(x_1,x_2)\,dx_1\right) & \text{in } \Omega, \\ u_\varepsilon = 0 & \text{on } \partial\Omega, \end{cases} \tag{2}$$

when $\varepsilon \to 0$.

Generalized Solutions for a Class of Nonlinear Diffusions

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A general class of nonlinear possibly degenerate diffusions will be introduced for which global existence of an appropriate weak solution can be shown. Examples and applications to Image Processing will discussed.

Positive and free boundary solutions for some nonlinear singular elliptic problems

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We prsent some recent results concerning existence and multipiicity of positve solutions for some quasilinear (and semilinear) elliptic problems with singular nonlinearities. In particular we can give a complete description of the solution set in the one- dimensional case.

A Partial-Integro Differential Equation (PIDE) for Sandpile

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Our aim is to introduce and study the Partial Integro-Differential Equation (PIDE) associated with the dynamic of some physical discrete structure, like sandpile. Our PIDE is closely related to the nonlocal evolution problem obtained by letting p goes to infinity in the nonlocal p-Laplacian equation. We also show the connection between our PIDE and the stochastic process introduced by Evans and Rezakhanlou for modeling sandpile problem.

On a nonlocal in time and space evolution equation

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First, a new technique to prove, in a general case, the recent result of Cazenave, Dickstein and Weissler on the blowing-up solutions to a temporally nonlocal nonlinear parabolic equation is presented. Then, the blow-up rate and the global existence in time of the solutions are presented. Furthermore, necessary conditions for global existence are considered. Keywords : Parabolic equation, local and global existence, critical exponent, fractional Laplacian, Riemann-Liouville fractional integrals and derivatives, blow-up rate. MSC : 35K55; 35B44; 35B33. Reference: T. Cazenave, F. Dickstein & F. D. Weissler, An equation whose Fujita critical exponent is not given by scaling, Nonlinear Analysis 68 (2008); 862 -874.

Image denoising using fractional time derivatives

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We propose a new method for the image denoising(restoration) by using a linear time fractional differential equation(equivalent to a second kind linear Volterra integral equation). The linear time fractional differential equation obtained by replacing ordinary time derivative in heat equation by Riemann-Liouville fractional derivative (order of derivative is between 1 and 2). The diffusion process has been controlled by the order of Riemann-Liouville fractional derivative, resulting a diffusion process which preserves the edges and structure of the image during denoising process. The proposed method is well-posed and admits a stable numerical discretization. Unlike the famous Perona-Malik model, discrete implementation is fully understood and don't lead to any paradox. Numerical examples are considered for the implementation of the proposed method. A comparison has been made with Perona-Malik model in terms of SNR and PSNR.

On some nonlinear anisotropic singular perturbation problems

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We consider a general framework of anisotropic singular perturbation problems depending on a parameter $\varepsilon \geq 0$. First we consider perturbed problems of the type $\varepsilon A u_\varepsilon + B u_\varepsilon = f$, where A and B are nonlinear operators defined on Banach spaces and we describe the asymptotic behavior of solutions u_ε when $\varepsilon \to 0$. We give some applications to boundary value problems where the parameter ε only affects some directions of the space variables. In this case the limit problem ($\varepsilon = 0$) is defined in lower dimension spaces. In the same framework we study the quasilinear elliptic problems of second order, with more investigation for the $p-$Laplacian problem.

On the analytic solutions of the Bratu Equations

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In this talk, analytic solution of the Bratu equation will be presented. It is based on the variational method which produces an implicit system of several variables. We used the predictor corrector technique to trace the solution curve generated from this system. Existence and non-uniqueness of solution is also discussed. The results presented here indicate that two solutions exist on the range $0 < \lambda < \lambda_c$, for some critical value λ_c, one solution exists for $\lambda = \lambda_c$, and no solution exists for$\lambda > \lambda_c$. An analytical value of the turning point of λ_c is also obtained. Numerical results and conclusions will be presented.

3 Rate of decay, Shrödinger operator, wave equation, fluid mechanics...

The wave equations with dynamic boundary conditions

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We investigate the wave equation with dynamic boundary conditions. Thanks to Galerkin approximation combined with the contraction mapping theorem, we prove a local existence result. Also, under appropriate restrictions on the initial data, we study the asymptotic behavior of our obtained solution (decay of solution, exponential growth, blow up).

A fluide-rigide interaction problem in lubrication

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A fluide-rigide interaction problem in lubrication We consider a lubricated system which consists in two rigid surfaces in hydrodynamic contact. The bottom surface, assumed planar and horizontal, moves with a constant horizontal velocity, while the upper surface can move only by vertical translation. The space between the two bodies is filled by an incompressible fluid. The pressure in the fluid satisfies a Reynolds variational inequality since the domain filled by the fluid is considered thin enough and the possible cavitation in the fluid must be taken into account. This problem is coupled with a second Newton for the displacemnt of the upper rigid body; we remark that this displacement appears as a parameter in the Reynolds variational inequality. We prove the existence of a global in time solution for this coupled system and we also prove that the distance between the rigid bodies is allways larger than a positive constant depending on the data of the problem.

Exponential decay for solutions to semilinear damped wave equation

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This talk is concerned with decay estimate of solutions to the semilinear wave equation with strong damping in a bounded domain. Introducing an appropriate Lyapunov function, we prove that when the damping is linear, we can find initial data, for which the solution decays exponentially. This result improves an early proved by Gazzola and Squassina, "Global solutions and finite time blow up for damped semilinear wave equations", Ann. I. H. Poincar, 23:185–207, 2006.

Similarity and pseudosimilarity solutions for a class of non-Newtonian fluids revisited

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The broad goal of this work is to re-examine a class of exact solutions for two-dimensional boundary-layer flows induced by continuous (permeable or impermeable) plane surfaces for Newtonian and non-Newtonian fluids (Ostwald-de Waele model). The stretching velocity is assumed to vary as x^m, $m = -\frac{1}{2n-1}$, where x is measured from the leading edge of the plate and $n \neq \frac{1}{2}$ is the power-law index.

From Stokes to Darcy in infinite cylinders : do limits commute ?

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Work in collaboration with Patrizia Donato from the University of Rouen and Bogdan Vernescu from the Worcester Polytechnic Institute. We study the flow of a fluid in a porous environment represented by a long cylinder containing periodically distributed small obstacles. More precisely, we make an asymptotic analysis of the flow of a fluid modeled by the Stokes equations, when the size of the obstacles goes to zero and the length of the cylinder goes to infinity. We combine in this way two already studied phenomena: the homogenization of the Stokes problem and the asymptotic analysis of the same problem in cylinders becoming unbounded in the direction of their axis. We study the question of commutativity of the two limits, but we also consider the simultaneous limit, i.e. we study the existence of a limit when the two quantities the size of the obstacles and the length of the cylinder - go to zero and infinity, respectively.

General decay of solutions of a wave equation with a boundary control of memory type

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In this paper we consider a nonlinear wave equation, in a bounded domain, where the memory-type damping is acting on a part of the boundary. We establish a general decay result, from which the usual exponential and polynomial decay rates are only special cases. Our work allows certain relaxation functions which are not necessarily of exponential or polynomial decay and, therefore, generalizes and improves earlier results in the literature.

Obesrvability and stabilization of a wave equation with dynamical control

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This work is concerned the boundary observability and the stabilization of a wave equation damped by dynamical control. First, using a frequency domain approach, we prove that the system is not uniformly stable and by a multiplier method, we establish the polynomial energy decay rate for smooth initial data. Next, a difficulty is due to the presence of the dynamical type in particular the observability inequalities obtained in the classical case does not hold. To overcome this difficulty, we reduce the space of the energy and we establish a new observability inequality.

On direct and inverse problems for the Schrdinger and Pauli operators

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We obtain new asymptotic formulae for the Bloch eigenvalues and Bloch function of the multidimensional Schrdinger operator with periodic potential corresponding to the quasimomentum lying near the diffraction hyperplane. Then using these formulas, we determine constructively new spectral invariants of this operator from the given Bloch eigenvalues. The new invariants are explicitly expressed by Fourier coefficients of the potential which present the possibility of determining the potential constructively by using the Bloch eigenvalues as input data. Then we give an algorithm for unique determination the potential by these new spectral invariants and hence by given Bloch eigenvalues. The approach used for the Shrdinger operator can be applied for solving the direct and inverse problems of Pauli operators and Shrdinger operator with magnetic and electric filds.

Eponential and polynomial stability of the nonuniform Timoshenko beam by one distributed feedback

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In this paper, we study the energy decay rate for a nonuniform Timoshenko beam. The system consists of two coupled wave equations. Only the equation about the rotation angle is damped by one locally distributed feedback at the neighborhood of the boundary. The equation for the transverse displacement of the beam is only indirectly damped through the coupling. First, we establish an exponential energy decay rate in the case of the same speed of propagation. Next, when the wave speeds are different, a polynomial type decay rate is obtained. These results are proved by verifying the frequency domain conditions.

4 Numerical analysis, viscoelasticity...

A Collocation-Shooting Method for Solving Fractional Boundary Value Problems

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In this paper, we discuss the numerical solution of special class of fractional boundary value problems of order 2. The method of solution is based on a conjugating Collocation and Spline analysis combined with shooting method. A theoretical analysis about the existence and uniqueness of exact solution for the present class is proven. Two examples involving Bagley-Torvik equation subject to boundary conditions are also presented; numerical results illustrate the accuracy of the present scheme.

Analytical Sequences of Upper and Lower Solutions for a Class of Elliptic Equations

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Comparison arguments are applied to derive decreasing sequences of upper solutions and increasing sequences of lower solutions for a class of nonlinear elliptic equations. The monotonicity of the two sequences is proven. These polynomial sequences are obtained by applying new algorithms and solving linear differential equations. The obtained bounds are analytic and have closed forms. Two examples are presented to explore the effectiveness of the new algorithms. The idea of the new algorithms can be extended to deal with different classes of equations.

On the stability of a nonlocal conservation law

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We are interested in a nonlocal conservation law which describes the dynamics of dunes. We investigate the stability of travelling-waves solutions. We prove that the constant travelling-wave is linearly unstable which means that basins with flat bathymetry do not exist in nature. We also propose numerical schemes to approximate the solutions of this equation and study their numerical stability. The continuous problem being linearly unstable for low frequency perturbation, we introduce a new definition of numerical stability for this nonlocal equation.

An existence theorem for the magneto-viscoelastic problem

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The dynamics of magneto-viscoelastic materials is described by a nonlinear system which couples the equation of the magnetization, given in Gibert form, and the integro-differential equation for the evolution of the viscoelastic displacements. We study the general three-dimensional case and establish, via compactness of the approximated penalty problem, a theorem for the existence of weak solutions. The crucial point in the proof is the characterization of some a-priori estimates which allow us to obtain convergence results. For this we need some constitutive assumptions on the viscoelastic kernel. The present investigation provides new insight on a study which originates in [1]-[4].

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Doubly nonlinear evolution equations of second order and their approximation

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A class of doubly nonlinear evolution equations of second order with a first-order damping term is studied. The operator acting on the zero-order term is assumed to be the sum of a linear, bounded, symmetric, strongly positive operator and a nonlinear operator that fulfills a certain growth and a Hölder-type continuity condition. The operator acting on the first-order time derivative is a nonlinear hemicontinuous operator that fulfills a certain growth condition and is (up to some shift) monotone and coercive. The convergence of a time discretisation on a variable time grid is shown by reducing the second-order equation to a parabolic integro-differential equation. This convergence also proves the existence of a weak solution.

On the existence of solutions of equilibria in lubricated journal

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We study a system of equations concerning equilibrium positions of journal bearings. The problem consists on two surfaces in relative motion separated by small distance filled with a lubricant. The shape of the inlet surface is circular while the other surface has a more general shape. In this talk we present results concerning the existence of at least one equilibrium.

On the numerical integration of one nonlinear parabolic equation

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In the present paper the difference analogue of the initial-boundary value problem to one nonlinear parabolic equation is considered. In certain conditions the convergence of the solution of the difference analogue to the solution of source problem is proved. For the same difference analogue the comparison theorem is proved and the uniqueness of the solution is obtained. The iteration process for finding of the solution of difference analogue is constructed and in certain conditions its convergence is obtained.

Computational Approach to Solve an Inverse Problem for Transport Equation

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This study presents a computational approach to the solution of a two space dimensional inverse problem for time-independent transport equation and the solvability of this inverse problem. To compute the approximate solution of the problem an algorithm is proposed and to demonstrate the computational feasibility of the given approximation method, some computational experiments are performed and the results are presented. The main difficulty of this study is the overdeterminacy of the problem. In the paper, using some extension of the class of unknown functions, the overdetermined inverse problem is replaced by a related determined one, which is an interesting technique of investigating the solvability.

Spatial asymptotic behavior in PDE and its applications

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We bring a short introduction to the spatial asymptotic estimates, then we formulate an estimate for finite elements and its application and then we bring results for variational inequalities with constraint on the gradient in cylinders and look at the two cases when the applied force is zero or not.

5 Variational methods...

A variational convergence for bifunctionals. Application to a model of strong junction

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We first introduce a notion of variational convergence for bifunctionals in an abstract setting. Then we apply this convergence to the asymptotic analysis of a junction problem in order to capture the gradient oscillations in the layer by considering the energy functional as a bifunctional of Sobolev-function/Young measure arguments. The well known asymptotic model described in terms of Sobolev-functions is obtained by eliminating the Young-measure argument considered as an internal variable through a marginal map. Furthermore the surface energy of the classical model can be considered as a relaxation of a Dirichlet condition.

Image by Fourier transform of some weighted Sobolev spaces

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In this talk, we expose a theorem which asserts that the image of a weighted Sobolev space is imbedded in a weighted space of the same form. Moreover, we prove that correspondance between the two spaces is a purely algebraic map, explicitly known. An infinity of weighted spaces which are invariant under Fourier transform is deduced.

On the quasi-convex of some functions in 2x2 dimensions without convexity assumptions

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In this note we would like to compute the quasiconvex envelope for a class of functions defined in 2x2 real matrices without convexity assumptions. This generalizes in particular some results contained in [Bo. Led]. [Bo. Led]M. Bousselsal and H. Ledret:Remarks on the quasi-convex envelopes for functions depending on quadratic forms. Bollettino UMI 5-B (2002) 469-486.

On a system of eikonal equations

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In this talk I will speak about a system of eikonal equations with a homogeneous Dirichlet boundary condition. Since this problem has infinitely many solutions, the question of selecting the ones having more regularity naturally arises. Our work goes in this direction; in particular we characterize a particular class of solutions through a variational method involving the discontinuity set of the gradient. The results which will be presented have been obtained in collaboration with G. Pisante.

Alternating proximal algorithm with costs-to-move, selective minimization and applications to variational problems and PDE's

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Alternating proximal algorithm with costs-to-move, selective minimization and applications to variational problems and PDE's In this talk, we study the alternating proximal algorithm with costs-to-move introduced by Attouch, Redont and Soubeyran. We study the convergence of the algorithm when the control parameter acting on the coupling term of the two involved variables is constant. Then we respectively replace this constant parameter by a sequence which increases towards infinity and by a sequence which decreases towards zero. In each case, we study the convergence of the algorithm and the hierarchical and selective minimization which appears, and we give an application to variational problems and EDP's.

Compatibility conditions for the fundamental theorems of Riemannian geometry by way of differential forms and shell theory

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Given a smooth enough field $\mathbf{C}$ of symmetric and positive-definite matrices of order three, satisfying a compatibility relation over a simply-connected open set $\Omega \subset \mathbf{R}^3$, then there exists, typically in spaces such as $W^{2,p}_{loc}(\Omega, \mathbf{R}^3)$, $p > 3$, or $\mathbf{C}^2(\Omega, \mathbf{R}^3)$, an immersion $\Theta : \Omega \to \mathbf{R}^3$ such that $\mathbf{C} = \nabla\Theta^T\nabla\Theta$ in Ω. We have at least two such compatibility relations. First the compatibility relation of the fundamental theorem of Riemannian geometry classically expresses that the Riemann curvature tensor associated with the field $\mathbf{C}$ vanishes in Ω. Second the relation (see[1]):

$$\text{CURL } \Lambda + \text{COF } \Lambda = 0 \text{ in } \Omega,$$

where the matrix field Λ is defined in terms of the field $\mathbf{U} = \mathbf{C}^{1/2}$ by

$$\Lambda = \frac{1}{det\mathbf{U}}\Big\{\mathbf{U}\ (\text{CURL } \mathbf{U})^T\mathbf{U} - \frac{1}{2}(\text{tr}[\mathbf{U}\ (\text{CURL } \mathbf{U})^T])\mathbf{U}\Big\}.$$

The main objective of this paper is to unify the two approaches in an intrinsic context of exterior differential forms (see for example [5] or [4]). The new proof is very elegant, simpler and uses a new form of Frobenius integration theorem with low regularity (see [6]). This approach constitutes one of the first steps towards the analysis of models in nonlinear three-dimensional elasticity, where the rotation field, which appears in the polar factorization of the deformation gradient, is considered as one of the primary unknowns (see for example [6]).
In addition, within the same theory we will unify the two sets of compatibility relations in the theory of surfaces, namely the Gauss and the Mainardi-Codazzi relations on the one hand and the Darboux-Vallée-Fortuné relations (see [8], [2], [3]) on the other hand. Such compatibility relations are an important tool in the modeling of intrinsic shells.

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Quasiconvexity at the boundary and weak lower semicontinuity

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We show a link between the notion of quasiconvexity at the boundary introduced by Ball and Mardsen as a necessary condition of a minimizer and weak lower semicontinuity of integral functionals. We also show that this notion plays a crucial role in a ful characterization of generalized Young measures generated by gradients.

On a Schaefer-Krasnoselskii type fixed point theorem for the weak topology In Banach spaces

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In this talk we present a Schaefer-Krasnoselskii type fixed point theorem for the sum of a continuous weakly compact operator A and a strict contraction B on general Banach spaces. The peculiarity of this result is that the weak continuity of operator A is not required. We close the talk by presenting an application from the theory of nonlinear integral equations in the context of L^1-spaces where, in general, we have a lack of compactness.

Nonlinear Korn inequalities and applications

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One version of Korn's inequality in a bounded domain D of the n-dimensional space $\mathbf{R}^n$ states that the distance in $L^2(D)$ from the gradient of a vector field $u : D \to \mathbf{R}^n$ to the set of all anti-symmetric matrices of order n is bounded from above, up to a multiplicative constant independent of u, by the $L^2(D)$-norm of the symmetric part of the gradient of u. In 2002, Friesecke, James, and Mller proved a nonlinear version of this inequality: The distance in $L^2(D)$ from the gradient of a vector field $u : D \to \mathbf{R}^n$ to the special orthogonal group of order n is bounded from above, up to a multiplicative constant independent of u, by the $L^2(D)$-norm of the function that at each point x of D associates the distance from the gradient of u at x to the special orthogonal group of order n. The talk presents some extensions and applications in elasticity of this result.