Title: "Global solutions for some reaction-diffusion systems: L^∞, L^p, L^1 and L^2-strategies

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Lots of reaction-diffusion systems arising in applications come with the two natural following properties:
Positivity of the solutions is preserved for all time
The total mass of the components is controlled for all time.
The fact that the total mass of the components does not blow up in finite time suggests that solutions should exist for all time (solutions are actually bounded in L^1, uniformly in time). But, it turns out that the answer is not so simple. An L^p-strategy provides global existence of classical solutions for a subfamily of these systems. But, to handle more of them, it is necessary to give up looking for bounded classical solutions and rather consider *weak solutions* which may be allowed to blow up in L^∞ at some time(s), and nevertheless continue to exist. For instance, global weak solutions exist as soon as the nonlinearities are bounded in L^1.
A curious L^2-estimate is a priori valid for all these systems. This estimate allows to prove global existence when nonlinearities are at most quadratic as it is the case for many chemical and biological systems. Actually, an a priori L^2-compactness even holds: it turns out to be an adequate tool to study the limit of chemical systems where some rate constants tend to infinity.
We will give a survey on these questions and describe some recent results together with open problems.