The system of coupled KdV equations: well-posedness and stability of solitary-wave solutions

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Consider here is the system

$$\begin{cases} u_t + u_{xxx} + P(u,v)_x = 0, \\ v_t + v_{xxx} + Q(u,v)_x = 0 \end{cases} \tag{0.1}$$

of coupled KdV-equations introduced by Bona, Cohen and Wang, where $u = u(x,t), v = v(x,t)$ are functions defined on $\mathbb{R} \times \mathbb{R}^+$, $P(u,v) = Au^2 + Buv + Cv^2$ and $Q(u,v) = Du^2 + Euv + Fv^2$ in which $A, B, \cdots, F$ are real number constants. My talk consists of two parts, the first is to show that the system is always locally well-posed in $H^s \times H^s$ for $s > -\frac{3}{4}$. Moreover, if system of linear equations

$$\begin{cases} 2Ba + (E-2A)b - 4Dc = 0, \\ 4Ca + (2F-B)b - 2Ec = 0 \end{cases} \tag{0.2}$$

has solutions (a,b,c) such that $4ac > b^2$, then (0.1) is globally well-posed in $H^s \times H^s$ for $s > -\frac{3}{4}$. Second part is to find out explicit solitary-wave solutions. This task can reduce to a cubic equation. There are up-to three solitary-wave solutions of hyperbolic square shape. Investigation of their stability tuns out to be very subtle and interesting.