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Stochastic variational inequalities with jumps

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We consider the following equation

$$dX_t + \partial\varphi\left(X_t\right)(dt) \ni b\left(X_t\right)dt + \sigma\left(X_t\right)dW_t + \int_{\mathbb{R}^d\setminus\{0\}} \gamma\left(X_{t-}, z\right)d\tilde{N}_t\left(dz\right),$$

where

- $\partial\varphi$ is the subdifferential of proper, l.s.c., convex function φ;
- W is a Brownian motion;
- $\tilde{N}$ is the compensated measure of a homogeneous Poisson random measure.

Lévy processes

Let $(\Omega, \mathcal{F}, P)$ be a complete probability space. A d-dimensional stochastic process $(L_t)_{t\geq 0}$ is called a *Lévy process* if:

1. it is càdlàg, *i.e.* $t \mapsto L_t$ is right continuous and has finite left limits a.s.;

2. it is stochastically continuous, *i.e.* $t \mapsto L_t$ is continuous in probability;

3. the random variables $L_{t_0}, L_{t_1} - L_{t_0}, \ldots, L_{t_n} - L_{t_{n-1}}$ are independent, for every $n \in \mathbb{N}^*$ and $0 \leq t_0 < t_1 < \cdots < t_n$;

4. $L_{t+s} - L_t$ has the same law as L_s, for every $t, s \geq 0$; in particular, $L_0 = 0$ a.s.

The law of a Lévy process $(L_t)_{t\geq 0}$ is determined by the *characteristic exponent* of L, *i.e.* the unique continuous function $\Psi : \mathbb{R}^d \to \mathbb{C}$ such that $\Psi(0) = 0$ and

$$\mathbb{E}\exp\left(i\langle \lambda, L_t\rangle\right) = \exp\left(-t\Psi(\lambda)\right), \; t \geq 0, \; \lambda \in \mathbb{R}^d.$$

By the Lévy-Khintchine formula for infinitely divisible distributions, Ψ has the following form:

$$\Psi(\lambda) = i\langle a, \lambda\rangle + \frac{1}{2}Q(\lambda) + \int_{\mathbb{R}^d\setminus\{0\}} \left(1 - e^{i\langle \lambda, x\rangle} + i\langle \lambda, x\rangle \mathbf{1}_{\{|x|<1\}}\right)\nu(dx), \tag{1}$$

where $a \in \mathbb{R}^d$, Q is a positive semi-definite quadratic form on $\mathbb{R}^d$, and ν is a measure on $\mathbb{R}^d\setminus\{0\}$ such that

$$\int \left(1 \wedge |x|^2\right)\nu(dx) < +\infty.$$

The measure ν is called the Lévy measure associated to L.

For every function Ψ given by the formula (1), there exists a Lévy process with characteristic exponent Ψ.

Examples:

- if $\Psi(\lambda) := \frac{1}{2}|\lambda|^2$, then L is a Brownian motion;
- if $\Psi(\lambda) := c\left(1 - \mathrm{e}^{i\lambda}\right)$, then L is a Poisson process of intensity $c > 0$;
- A generalization of a Poisson process is the following:

Let $\xi_1, \ldots, \xi_n, \ldots$ be independent random variables with the same distribution ν on $\mathbb{R}^d \setminus \{0\}$, and

$$S(n) := \xi_1 + \cdots + \xi_n$$

the corresponding random walk. If $(N_t)_{t \geq 0}$ is a Poisson process of intensity $c > 0$, then the process

$$S \circ N_t = \sum_{i=1}^{N_t} \xi_i$$

is a Lévy process, called a compound Poisson process with Lévy measure $c\nu$.

The characteristic exponent of $S \circ N_t$ is

$$\psi(\lambda) = c \int_{\mathbb{R}^d} \left(1 - e^{i\langle\lambda, x\rangle}\right) \nu(dx);$$

so, every Lévy process with a finite Lévy measure can be represented as the sum of a Brownian motion and an independent compound Poisson process.

Definition. Let ν be a σ-finite measure on $\mathbb{R}^d$. A random measure $N(\omega, dt, dz)$ is a *homogeneous Poisson random measure* with intensity ν, if:

i) for each $\omega \in \Omega$, $N(\omega, ., .)$ is a measure on $\mathbb{R}_+ \times \mathbb{R}^d$;

ii) for each set $B \in \mathcal{B}(\mathbb{R}_+ \times \mathbb{R}^d)$ with $(dt \otimes \nu)(B) < +\infty$, the random variable $N(\cdot, B)$ is Poisson with parameter $(dt \otimes \nu)(B)$;

iii) if $B_1, \ldots, B_n$ are disjoint Borel sets of $\mathbb{R}_+ \times \mathbb{R}^d$, then $N(\cdot, B_1), \ldots, N(\cdot, B_n)$ are independent.

The *compensated random measure* of N

$$\tilde{N}(dt, dz) := N(dt, dz) - dt \otimes \nu(dz)$$

has the property that $t \longmapsto \tilde{N}([0, t], A)$ is a martingale for every $A \in \mathcal{B}(\mathbb{R}^d)$ with $\nu(A) < +\infty$.

If L is a Lévy process on $\mathbb{R}^d$ with Lévy measure ν, then its *jump counting measure*, defined by

$$N\left(t, A\right) := \sum_{0<s\leq t} \mathbf{1}_A\left(\Delta L_s\right),\ t>0,\ A\in\mathcal{B}(\mathbb{R}^d) \text{ with } 0\notin\overline{A},$$

is a homogeneous Poisson random measure with intensity ν, and the following holds:

$$L_t = bt + \sigma W_t + \int_{\{|z|<1\}} z\tilde{N}_t\left(dz\right) + \int_{\{|z|\geq 1\}} zN_t\left(dz\right), \tag{2}$$

where $b\in\mathbb{R}^d$, $\sigma\in\mathbb{R}^{d\times d}$, and W is a Brownian motion independent of N.

Jump-diffusions

Jump-diffusions are generalizations of SDEs driven by Lévy processes. Let $(\Omega, \mathcal{F}, P)$ be a complete probability space endowed with a right-continuous, complete filtration $\mathbb{F} = \{\mathcal{F}_t\}_{t\geq 0}$, W a d'-dimensional $\mathbb{F}$-Brownian motion, and N a Poisson random measure with intensity ν, $\mathbb{F}$-adapted and independent of W. Let us consider $b : \mathbb{R}^n \to \mathbb{R}^n$, $\sigma : \mathbb{R}^n \to \mathbb{R}^{n\times d'}$, $\gamma : \mathbb{R}^n \times \left(\mathbb{R}^d \backslash \{0\}\right) \to \mathbb{R}^n$ satisfying the following assumptions:

(H1) b and σ are Lipschitz functions;

(H2) γ is a measurable function with

$$\int |\gamma(0,z)|^2 \nu(dz) < +\infty \text{ and } \int |\gamma(x,z) - \gamma(x',z)|^2 \nu(dz) \leq L |x - x'|^2 .$$

Theorem (Gihman, Skorohod, 1972). Under the above assumptions, equation

$$dX_t = b(X_t)\,dt + \sigma(X_t)\,dW_t + \int_{\mathbb{R}^d \backslash \{0\}} \gamma(X_{t-}, z)\,d\tilde{N}_t(dz)$$

has a unique solution starting at $\xi \in L^2(\Omega, \mathcal{F}_0, P)$.

Reflected jump-diffusions

Let O be an open, convex and bounded subset of $\mathbb{R}^n$. Under assumptions (H1),

(H2)' for every $p \geq 2$, we have

$$\int |\gamma(0,z)|^p \nu(dz) \leq C_p \text{ and } \int |\gamma(x,z) - \gamma(x',z)|^p \nu(dz) \leq C_p |x - x'|^p .$$

and

(H3) $x + \gamma(x,z) \in \overline{O}, \ \forall x \in \overline{O}$,

Menaldi, Robin (1985) proved that equation

$$dX_t = b(X_t)\,dt + \sigma(X_t)\,dW_t + \int_{\mathbb{R}^d \setminus \{0\}} \gamma(X_{t-}, z)\,d\tilde{N}_t(dz) - dK_t,$$

where K is the reflecting process of X on the boundary of O, admits a unique solution with given initial starting point $x \in \overline{O}$.

Variational inequalities

Let $\varphi : \mathbb{R}^n \to \overline{\mathbb{R}}$ be a proper, l.s.c., convex function with $\operatorname{int}(\operatorname{Dom}\varphi) \neq \emptyset$. The subdifferential of φ is defined by

$$\partial\varphi(x) := \{x^* \in \mathbb{R}^n \mid \langle x^*, y - x\rangle + \varphi(x) \leq \varphi(y), \ \forall y \in \mathbb{R}^n\}.$$

In the case $\varphi \equiv I_{\overline{O}} : x \mapsto \begin{cases} 0, & x \in \overline{O}; \\ +\infty, & x \notin \overline{O}, \end{cases}$ the subdifferential is given by

$$\partial I_{\overline{O}}(x) = \begin{cases} \{0\}, & x \in O; \\ N_{\overline{O}}(x), & x \in \operatorname{bd} O; \\ \emptyset, & x \notin \overline{O}. \end{cases}$$

We consider the following equation

$$dX_t + \partial\varphi(X_t)\,dt \ni b(X_t)\,dt + \sigma(X_t)\,dW_t + \int_{\mathbb{R}^d\setminus\{0\}} \gamma(X_{t-}, z)\,d\tilde{N}_t(dz), \tag{3}$$

We denote by $D([0,T];\mathbb{R}^n)$ the class of $\mathbb{R}^n$-valued, càdlàg functions on $[0,T]$, endowed with the uniform convergence topology. We say that $(X, K) \in L^2_{\text{ad}}(\Omega; D([0,T];\mathbb{R}^n)) \times L^2_{\text{ad}}(\Omega; C([0,T];\mathbb{R}^n))$ is a solution of (3) if:

- $\varphi(X) \in L^1(\Omega \times [0,T])$;
- $K \in L^1(\Omega; BV_0([0,T];\mathbb{R}^n))$;
- $X_t + K_t = \int_0^t b(X_s)\,ds + \int_0^t \sigma(X_s)\,dW_s + \int_0^t \int_{\mathbb{R}^d\setminus\{0\}} \gamma(X_{s-}, z)\,d\tilde{N}_s(dz)$;
- $\int_0^T \langle y(r) - X_r, dK_r\rangle + \int_0^T \varphi(X_r)\,dr \le \int_0^T \varphi(y(r))\,dr$, $\forall y \in D([0,T];\mathbb{R}^n)$.

Asiminoaiei, Răşcanu (1997): case $\gamma \equiv 0$.

Theorem (Uniqueness). Under assumptions (H1), (H2), equation (3) has at most one solution starting from $x \in \operatorname{Dom} \varphi$.

For the proof, we consider two solutions (X, K) and $(\tilde{X}, \tilde{K})$ and apply Itô's formula to $\left|X_t - \tilde{X}_t\right|^2$:

$$\begin{aligned}
&\left|X_t - \tilde{X}_t\right|^2 + \int_0^t \left\langle X_s - \tilde{X}_s, d(K_s - \tilde{K}_s)\right\rangle \\
&= 2\int_0^t \left\langle X_s - \tilde{X}_s, b\left(X_s\right) - b(\tilde{X}_s)\right\rangle ds + 2\int_0^t \left\langle X_s - \tilde{X}_s, \left[\sigma\left(X_s\right) - \sigma(\tilde{X}_s)\right] dW_s\right\rangle \\
&+ 2\int_0^t \int_{\mathbb{R}^d \setminus \{0\}} \left\langle X_{s-} - \tilde{X}_{s-}, \gamma\left(X_{s-}, z\right) - \gamma(\tilde{X}_{s-}, z)\right\rangle + \left|\gamma\left(X_{s-}, z\right) - \gamma(\tilde{X}_{s-}, z)\right|^2 d\tilde{N}_s\left(dz\right) \\
&+ \int_0^t \left|\sigma\left(X_s\right) - \sigma(\tilde{X}_s)\right|^2 ds + \int_0^t \int_{\mathbb{R}^d \setminus \{0\}} \left\langle X_{s-} - \tilde{X}_{s-}, \gamma\left(X_{s-}, z\right) - \gamma(\tilde{X}_{s-}, z)\right\rangle \nu\left(dz\right) ds.
\end{aligned}$$

For the existence result, we impose the condition $(q \geq 1)$

(H3)' $\varphi(x + \gamma(x,z)) \leq \varphi(x) + C_\gamma (1 + |\gamma(x,z)|^q), \ \forall (x,z) \in \mathbb{R}^n \times (\mathbb{R}^d \setminus \{0\})$.

Theorem (Existence). Under assumptions (H1), (H2)', (H3)', equation (3) has a unique solution starting from $x_0 \in \operatorname{Dom} \varphi$.

The proof of this result uses the penalization method. We consider the Yosida regularization of φ

$$\varphi_\varepsilon(x) := \inf \left\{ \frac{1}{2\varepsilon} |x - y|^2 + \varphi(y) \mid y \in \mathbb{R}^n \right\}, \ \varepsilon > 0,$$

which is a C^1, convex function on $\mathbb{R}^n$, with $\nabla \varphi_\varepsilon$ a Lipschitz function with Lipschitz constant equal to $1/\varepsilon$. Moreover, by (H3)',

$$\varphi_\varepsilon(x + \gamma(x,z)) \leq \varphi_\varepsilon(x) + C_\gamma (1 + |\gamma(x,z)|^q);$$

also, for simplicity, we can assume that $\varphi(x) \geq \varphi(0) = 0, \ \forall x \in \mathbb{R}^n$ and $0 \in \operatorname{int}(\operatorname{Dom} \varphi)$.

We consider the jump-diffusion X^ε given by

$$dX_t^\varepsilon + \nabla \varphi_\varepsilon(X_t^\varepsilon) dt = b(X_t^\varepsilon) dt + \sigma(X_t^\varepsilon) dW_t + \int_{\mathbb{R}^d \setminus \{0\}} \gamma(X_{t-}^\varepsilon, z) d\tilde{N}_t(dz).$$

We will show that X^ε and $K_t^\varepsilon := \int_0^t \nabla \varphi_\varepsilon(X_s^\varepsilon) ds$ converge to X and K.

I. Boundedness of (X^ε) and (K^ε)

Itô's formula for $|X_t^\varepsilon|^2$

$$
\begin{aligned}
|X_t^\varepsilon|^2 + 2\int_0^t \langle X_s^\varepsilon, \nabla\varphi_\varepsilon(X_s^\varepsilon)\rangle ds &= |x_0|^2 + \int_0^t [2\langle X_s^\varepsilon, b(X_s^\varepsilon)\rangle + |\sigma(X_s^\varepsilon)|^2] ds \\
&+ \int_0^t \int_{\mathbb{R}^d\setminus\{0\}} |\gamma(X_s^\varepsilon, z)|^2 \nu(dz)\, ds + 2\int_0^t \langle X_s^\varepsilon, \sigma(X_s^\varepsilon)\, dW_s\rangle \\
&+ \int_0^t \int_{\mathbb{R}^d\setminus\{0\}} \left[|X_{s-}^\varepsilon + \gamma\left(X_{s-}^\varepsilon, z\right)|^2 - \left|X_{s-}^\varepsilon\right|^2\right] d\tilde{N}_s(dz)\,.
\end{aligned}
$$

We obtain, since $\varphi_\varepsilon(x) \le \langle \nabla\varphi_\varepsilon(x), x\rangle,\ \forall x \in \mathbb{R}^n$,

$$
\mathbb{E}\sup_{t\in[0,T]} |X_t^\varepsilon|^4 \vee \mathbb{E}\left(\int_0^T \varphi_\varepsilon(X_s^\varepsilon)\, ds\right)^2 \le C\left(1 + |x_0|^4\right).
$$

Also, $\exists r_0 > 0,\ \exists M_0 > 0 : r_0 |\nabla\varphi_\varepsilon(x)| \le \langle x, \nabla\varphi_\varepsilon(x)\rangle + M_0,\ \forall x \in \mathbb{R}^n$; it follows that

$$
\mathbb{E}\,\|K^\varepsilon\|^2_{BV([0,T];\mathbb{R}^n)} = \mathbb{E}\left(\int_0^T |\nabla\varphi_\varepsilon(X_s^\varepsilon)|\, ds\right)^2 \le C\left(1 + |x_0|^4\right).
$$

II. Estimate for $\mathbb{E}\left[\sup_{t\in[0,T]} |\nabla\varphi_\varepsilon(X_t^\varepsilon)|^4\right]$

Itô's formula for $\varphi_\varepsilon^2(X_t^\varepsilon)$:

$$\begin{aligned}
\varphi_\varepsilon^2(X_t^\varepsilon) + \int_0^t \varphi_\varepsilon(X_s^\varepsilon)|\nabla\varphi_\varepsilon(X_s^\varepsilon)|^2 ds &\le \varphi_\varepsilon(x_0)^2 + 2\int_0^t \varphi_\varepsilon(X_s^\varepsilon)\langle\nabla\varphi_\varepsilon(X_s^\varepsilon), b(X_s^\varepsilon)\rangle ds \\
&+ \int_0^t |\nabla\varphi_\varepsilon(X_s^\varepsilon)|^2 |\sigma(X_s^\varepsilon)|^2 ds + \frac{1}{\varepsilon}\int_0^t \varphi_\varepsilon(X_s^\varepsilon)|\sigma(X_s^\varepsilon)|^2 ds \\
&+ \int_0^t \varphi_\varepsilon(X_s^\varepsilon)\langle\nabla\varphi_\varepsilon(X_s^\varepsilon), \sigma(X_s^\varepsilon)dW_s\rangle \\
&+ \int_0^t \int_{\mathbb{R}^d\setminus\{0\}} \mathcal{D}^0_{\varphi_\varepsilon^2}\left(X_{s-}^\varepsilon; \gamma\left(X_{s-}^\varepsilon, z\right)\right) d\tilde{N}_s(dz) \\
&+ \int_0^t \int_{\mathbb{R}^d\setminus\{0\}} \mathcal{D}^1_{\varphi_\varepsilon^2}(X_s^\varepsilon; \gamma(X_s^\varepsilon, z))\,\nu(dz)\,ds,
\end{aligned}$$

where

$$\begin{aligned}
\mathcal{D}_f^0(x;a) &:= f(x+a) - f(x); \\
\mathcal{D}_f^1(x;a) &:= f(x+a) - f(x) - \langle\nabla f(x), a\rangle.
\end{aligned}$$

We have, since $|\nabla\varphi_\varepsilon(x)|^2 \le \frac{2}{\varepsilon}\varphi_\varepsilon(x)\,,\ \forall x\in\mathbb{R}^n$,

$$
\begin{aligned}
\left|\mathcal{D}^0_{\varphi^2_\varepsilon}\left(X^\varepsilon_{s-};\gamma\left(X^\varepsilon_{s-},z\right)\right)\right| &\le \sup_{\mu\in[0,1]} 2\left|\nabla\varphi_\varepsilon\left(X^\varepsilon_{s-}+\mu\gamma\left(X^\varepsilon_{s-},z\right)\right)\right|\left|\varphi_\varepsilon\left(X^\varepsilon_{s-}+\mu\gamma\left(X^\varepsilon_{s-},z\right)\right)\right|\left|\gamma\left(X^\varepsilon_{s-},z\right)\right| \\
&\le \frac{2\sqrt{2}}{\varepsilon^{1/2}}\sup_{\mu\in[0,1]}\left|\varphi_\varepsilon\left(X^\varepsilon_{s-}+\mu\gamma\left(X^\varepsilon_{s-},z\right)\right)\right|^{3/2}\left|\gamma\left(X^\varepsilon_{s-},z\right)\right| \\
&\le \frac{2\sqrt{2}}{\varepsilon^{1/2}}\left|\left[\varphi_\varepsilon(X^\varepsilon_{s-})+C\left(1+\left|\gamma\left(X^\varepsilon_{s-},z\right)\right|^q\right)\right]^{3/2}\left|\gamma\left(X^\varepsilon_{s-},z\right)\right|; \\
\left|\mathcal{D}^1_{\varphi^2_\varepsilon}\left(X^\varepsilon_{s};\gamma\left(X^\varepsilon_{s},z\right)\right)\right| &\le \left(\sup_{\mu\in[0,1]}\left[\left|\nabla\varphi_\varepsilon\left(X^\varepsilon_{s}+\mu\gamma\left(X^\varepsilon_{s-},z\right)\right)\right|^2\right]+\frac{1}{\varepsilon}\varphi_\varepsilon\left(X^\varepsilon_s\right)\right)\left|\gamma\left(X^\varepsilon_{s-},z\right)\right|^2 \\
&\le \frac{2}{\varepsilon}\sup_{\mu\in[0,1]}\left[\varphi_\varepsilon\left(X^\varepsilon_{s}+\mu\gamma\left(X^\varepsilon_{s-},z\right)\right)\right]\left|\gamma\left(X^\varepsilon_{s-},z\right)\right|^2 \\
&\le \frac{2}{\varepsilon}\left[\varphi_\varepsilon(X^\varepsilon_s)+C\left(1+\left|\gamma\left(X^\varepsilon_{s-},z\right)\right|^q\right)\right]\left|\gamma\left(X^\varepsilon_{s-},z\right)\right|^2.
\end{aligned}
$$

This will give the following estimate:

$$
\mathbb{E}\left[\sup_{t\in[0,T]}\left|\nabla\varphi_\varepsilon\left(X^\varepsilon_t\right)\right|^4\right] \le \frac{C}{\varepsilon^{7/2}}\left(1+|x_0|^{q'}+\varphi\left(x_0\right)^2\right).
$$

III. Cauchy sequences argument

Itô's formula for $\left|X^{\varepsilon}-X^{\delta}\right|^{2}$:

$$\begin{aligned}
&\left|X_t^{\varepsilon}-X_t^{\delta}\right|^2+2\int_0^t\langle X_s^{\varepsilon}-X_s^{\delta},\nabla\varphi_{\varepsilon}\left(X_s^{\varepsilon}\right)-\nabla\varphi_{\delta}\left(X_s^{\delta}\right)\rangle ds\\
&=\int_0^t[2\langle X_s^{\varepsilon}-X_s^{\delta},b\left(X_s^{\varepsilon}\right)-b\left(X_s^{\delta}\right)\rangle+\left|\sigma\left(X_s^{\varepsilon}\right)-\sigma\left(X_s^{\delta}\right)\right|^2]ds\\
&+\int_0^t\int_{\mathbb{R}^d\setminus\{0\}}|\gamma\left(X_s^{\varepsilon},z\right)-\gamma\left(X_s^{\delta},z\right)|^2\nu\left(dz\right)ds\\
&+2\int_0^t\langle X_s^{\varepsilon}-X_s^{\delta},(\sigma\left(X_s^{\varepsilon}\right)-\sigma\left(X_s^{\delta}\right))dW_s\rangle\\
&+\int_0^t\int_{\mathbb{R}^d\setminus\{0\}}\left[|X_{s-}^{\varepsilon}-X_{s-}^{\delta}+\gamma\left(X_{s-}^{\varepsilon},z\right)-\gamma\left(X_{s-}^{\delta},z\right)|^2-\left|X_{s-}^{\varepsilon}-X_{s-}^{\delta}\right|^2\right]d\tilde{N}_s\left(dz\right).
\end{aligned}$$

We use the fact that

$$\langle\nabla\varphi_{\varepsilon}\left(x\right)-\nabla\varphi_{\delta}\left(y\right),x-y\rangle\geq-\left(\varepsilon+\delta\right)\langle\nabla\varphi_{\varepsilon}\left(x\right),\nabla\varphi_{\delta}\left(y\right)\rangle$$

to obtain that $\mathbb{E}\sup_{t\in[0,T]}\left|X_t^{\varepsilon}-X_t^{\delta}\right|^4\to 0$ as $\delta,\varepsilon\to 0$ and $\mathbb{E}\sup_{t\in[0,T]}\left|K_t^{\varepsilon}-K_t^{\delta}\right|^2\to 0$.

IV. Passing to the limit

There exist $X \in L^4_{\mathrm{ad}}(\Omega; D([0,T];\mathbb{R}^n))$ and $K \in L^2_{\mathrm{ad}}(\Omega; C([0,T];\mathbb{R}^n))$ such that

$$\mathbb{E} \sup_{t\in[0,T]} \left[|X^\varepsilon_t - X_t|^4 + |K^\varepsilon_t - K_t|^2 \right] \to 0$$

Since (K^ε) is also bounded in $L^2(\Omega; BV_0([0,T];\mathbb{R}^n))$, it converges weakly to K. It is now a standard argument to show that (X, K) is a solution of equation (3).

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Thank you for your attention!