Non-local elasto-viscoplastic models with dislocations

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1. Based on the existence of configurations with torsion
   $(\exists)\mathcal{K}_t \equiv \mathcal{K}$ config. with torsion $\iff$
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$$\psi = \psi_{\mathcal{K}}(\mathbf{C}^e, \overset{(e)}{\mathcal{A}}_{\mathcal{K}}, (\mathbf{F}^p)^{-1}, \overset{(p)}{\mathcal{A}}_{\mathcal{K}}, \rho^d_{\mathcal{K}}, \nabla_{\mathcal{K}}\rho^d_{\mathcal{K}}) \tag{1}$$

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as a function dependent on

- the second order elastic deformation $(\mathbf{C}^e, \overset{(\mathrm{e})}{\mathcal{A}}_{\mathcal{K}})$
- the plastic measure of deformation $((\mathbf{F}^p)^{-1}, \overset{(\mathrm{p})}{\mathcal{A}}_{\mathcal{K}})$
- the scalar density of dislocation $\rho^d_{\mathcal{K}}$ and its gradient

Imbalanced free energy

Ax. The virtual internal power in $\mathcal{K}$

$$virt(\mathcal{P}_{int})_{\mathcal{K}} = \frac{1}{\rho}(\mathbf{T} + \mathbf{T}^*) \cdot \widetilde{\mathbf{L}}^e + \frac{1}{\rho_{\mathcal{K}}} \boldsymbol{\mu}_{\mathcal{K}} \cdot virt\mathcal{L}_{\mathbf{L}^p}[\overset{(e)}{\mathcal{A}}_{\mathcal{K}}] +$$

$$+ \frac{1}{\rho_{\mathcal{K}}} \boldsymbol{\Upsilon}^p_{\mathcal{K}} \cdot \widetilde{\mathbf{L}}^p + \frac{1}{\rho_{\mathcal{K}}} \boldsymbol{\mu}^p_{\mathcal{K}} \cdot \nabla_{\mathcal{K}} \widetilde{\mathbf{L}}^p + \frac{1}{\rho_{\mathcal{K}}} \boldsymbol{\Upsilon}^{\lambda}_{\mathcal{K}} \cdot (\delta \boldsymbol{\Lambda}) +$$

$$+ \frac{1}{\rho_{\mathcal{K}}} \Upsilon^d_{\mathcal{K}} \cdot \delta \rho^d_{\mathcal{K}} + \frac{1}{\rho_{\mathcal{K}}} \mu^d_{\mathcal{K}} \cdot \nabla_{\mathcal{K}} \delta \rho^d_{\mathcal{K}}.$$

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$$+\frac{1}{\rho_{\mathcal{K}}}\boldsymbol{\Upsilon}^p_{\mathcal{K}}\cdot\widetilde{\mathbf{L}}^p + \frac{1}{\rho_{\mathcal{K}}}\boldsymbol{\mu}^p_{\mathcal{K}}\cdot\nabla_{\mathcal{K}}\widetilde{\mathbf{L}}^p + \frac{1}{\rho_{\mathcal{K}}}\boldsymbol{\Upsilon}^{\lambda}_{\mathcal{K}}\cdot(\delta\boldsymbol{\Lambda})+$$

$$+\frac{1}{\rho_{\mathcal{K}}}\Upsilon^d_{\mathcal{K}}\cdot\delta\rho^d_{\mathcal{K}} + \frac{1}{\rho_{\mathcal{K}}}\mu^d_{\mathcal{K}}\cdot\nabla_{\mathcal{K}}\delta\rho^d_{\mathcal{K}}.$$

Ax. The elasto-plastic behavior of the material is restricted to satisfy in $\mathcal{K}$ the imbalanced free energy condition

$$-\dot{\psi}_{\mathcal{K}} + (\mathcal{P}_{int})_{\mathcal{K}} \geq 0 \quad \text{for any virtual (isothermic) processes.} \qquad (2)$$

Micro stress and stress momentum

- obey the balance equations for micro forces:

$$\boldsymbol{\Upsilon}^p_{\mathcal{K}} = \text{div}\,(\boldsymbol{\mu}^p_{\mathcal{K}} - \boldsymbol{\mu}_{\mathcal{K}}) + \tilde{\rho} B^p_m, \quad \text{in } \mathcal{K}(\mathcal{P}, t), \quad \text{plastic micro forces}$$

$$\Upsilon^d_{\mathcal{K}} = \text{div}_{\mathcal{K}}\, \boldsymbol{\mu}^d_{\mathcal{K}} + \tilde{\rho} B^d_m, \quad \text{associated with the scalar dislocations}$$

- satisfy the viscoplastic type constitutive equations

Thermomechanics restrictions

are derived from the energy imbalance:

- Elastic type constitutive equations
- Evolution equations for $\mathcal{K}_t -$ have to be compatible with the dissipation inequlity.

Model with couple stresses and plastic connection:

- S. C-T., *ZAMP* **53** (2002)
- S. C-T., *Theoret. Appl. Mech.* **28** (2002)
- S. C-T., *in Continuum Models and Discret Systems*, Eds. Bergman & Inan (2004)
- S. C-T., *in Configurational Mech.*, Eds. Kalpakides and Maugin (2004)
- S. C-T., *Int.J. Fracture* (2007)
- S. C-T., *in Material Forces*, Eds. Steinmann and Maugin (2009)
- S. C-T., *Int.J. Fracture* (2010)
- S. C-T., V. Tigoiu, *J. Damage Mech.* (2010)

- The paper deals with a special case of elasto-plastic models within the crystal plasticity framework, i.e. no gradient of plastic distorsion is involved.
- The behaviour of the material is linear elastic with respect to plastically deformed configuration, without macro stress momnetum.
- The complet set of evolution equations has been derived, following Bortoloni and Cermelli (2004).
- The initial and boundary value problem for elasto-vascoplastic model with non-local equation for the scaler dislocation densities has been derived.

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- The complet set of evolution equations has been derived, following Bortoloni and Cermelli (2004).
- The initial and boundary value problem for elasto-vascoplastic model with non-local equation for the scaler dislocation densities has been derived.
- As applications in the paper by Bortoloni and Cermelli (2004):
  - the elastic behaviour is neglected
  - the shear stress is kept constant.
  - only one slip system is activated.

References

- Bortoloni L., Cermelli, P. *Journal of Elasticity* **76** (2004).
- Cermelli, P., Gurtin M. E. *Int. J. Solids Struc.* **39** (2002).
- Gurtin M. E., Needleman A. *Int. J. Solids Struc.* **53** (2005).
- S. C-T *Eur. J. Mech., A/Solids* **15** (1996).
- S. C-T *Int. J. Plast.* (2001)
- S. C-T, Soós, E. *Appl.Mech.Rev.* **43** (1990).
- S. C-T *Int. J. Engng. Sc.* **28** (1990) 171-191, 273-284.
- Mandel J. *Plasticité classique et viscoplasticité*
- Simo J.C., Hughes T.J.R. *Computational Plasticity* (2000).
- Teodosiu C., Raphanel J.L. in *Large Plastic Deformation* MECAMAT'91 (1993)

Elastic response

- Stress measures: $\mathbf{T}-$ Cauchy
- $\mathbf{S}-$ first Piola-Kirchhoff in ref. config.
- $\mathbf{\Pi}-$ second Piola-Kirchhoff in relax. config.

$$\begin{aligned} &\mathbf{T} = det\mathbf{F}^e(\mathbf{F}^e)^{-1}\mathbf{\Pi}(\mathbf{F}^e)^{-T}, \\ &\mathbf{S}(\mathbf{F}^p)^T = (det\mathbf{F}^p)\mathbf{F}^e\mathbf{\Pi} \\ &\quad \text{with} \quad \hat{\rho}\, det\mathbf{F}^e = \tilde{\rho}, \quad \tilde{\rho}(det\mathbf{F}^p) = \hat{\rho}_0 \end{aligned} \tag{4}$$

$\tilde{\rho}, \hat{\rho}\ \tilde{\rho}_0-$ mass densities.

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$\tilde{\rho}, \hat{\rho}\ \tilde{\rho}_0-$ mass densities.

- elastic type constitutive equation (if $\exists$ the potential φ.)

$$\mathbf{\Pi} = \tilde{\rho}\frac{\partial\varphi(\mathbf{E}^e)}{\partial\mathbf{E}^e}, \qquad \mathbf{E}^e = \frac{1}{2}(\mathbf{F}^{e\,T}\mathbf{F}^e - \mathbf{I}) \quad \text{relax. config.}$$

$$\mathbf{T} = \hat{\rho}\mathbf{F}^e\frac{\partial\varphi(\mathbf{E}^e)}{\partial\mathbf{E}^e}(\mathbf{F}^e)^T \quad \text{def. config.} \tag{5}$$

$$\mathbf{S} = \hat{\rho}_0\ \mathbf{F}^e\frac{\partial\varphi(\mathbf{E}^e)}{\partial\mathbf{E}^e}(\mathbf{F}^p)^{-T}, \qquad \text{ref. config.}$$

- linear momentum balance equation

$$div\mathbf{T} = 0 \quad \text{in} \quad \chi(\mathcal{B}, t)$$
$$div\mathbf{S} = 0 \quad \text{in} \quad \mathcal{B} \qquad (6)$$

when the boby forces are neglected.

- angular momentum balance equation

$$\mathbf{T} = \mathbf{T}^T \quad \Longleftrightarrow \quad \mathbf{F}\mathbf{S}^T = \mathbf{S}\mathbf{F}^T \qquad (7)$$

Rate of plastic distortion

- evolution equation for the plastic distortion

$$\dot{\mathbf{F}}^p(\mathbf{F}^p)^{-1} = \sum_{\alpha=1}^{N} \nu^\alpha \bar{\mathbf{s}}^\alpha \otimes \bar{\mathbf{m}}^\alpha \qquad (8)$$

with N- the number of the slip-systems,
$(\bar{\mathbf{s}}^\alpha, \bar{\mathbf{m}}^\alpha)$ the $\alpha-$ slip system,
$\bar{\mathbf{m}}^\alpha-$ normal to the slip plane, $\bar{\mathbf{s}}^\alpha-$ and $\bar{\mathbf{s}}^\alpha \cdot \bar{\mathbf{m}}^\alpha = 0$.
$\nu^\alpha = \dot{\gamma}^\alpha-$ slip velocity
$\gamma^\alpha-$ plastic shear on $\alpha-$ slip system.

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  $\nu^\alpha = \dot{\gamma}^\alpha-$ slip velocity
  $\gamma^\alpha-$ plastic shear on $\alpha-$ slip system.
- $(\bar{\mathbf{s}}^\alpha, \bar{\mathbf{m}}^\alpha)$ of the $\alpha-$ slip system are considered to be fixed with respect to the reference configuration (see Mandel (1971) and Teodosiu (1976))

viscoplastic flow

- is described by:

$$\dot{\gamma}^{\alpha} = \dot{\gamma}_0 \left| \frac{\tau^{\alpha}}{\zeta^{\alpha}} \right|^{n} \operatorname{sgn} \tau^{\alpha} H(\mathcal{F}^{\alpha}), \quad \nu^{\alpha} \equiv \dot{\gamma}^{\alpha} \quad \forall \alpha = 1, \dots, N, \tag{9}$$

$H(\mathcal{F}^{\alpha})$ is the Heaviside function composed with

- the activation function $\mathcal{F}^{\alpha}$

$$\mathcal{F}^{\alpha} := |\tau^{\alpha}| - \zeta^{\alpha} \tag{10}$$

$\tau^{\alpha}-$ resolved shear stress
$\zeta^{\alpha}-$ yield modulus (critical shear stress).

- via the Schmid's law

$$|\tau^{\alpha}| \geq \zeta^{\alpha} \Leftrightarrow \mathcal{F}^{\alpha} := |\tau^{\alpha}| - \zeta^{\alpha} \geq 0 \tag{11}$$

resolved shear stress

- is defined in terms of **T** or **S**

$$\tau^\alpha := \frac{\mathbf{T}}{\hat{\rho}}\mathbf{m}^\alpha \cdot \mathbf{s}^\alpha, \quad \mathbf{s}^\alpha = \mathbf{F}^e\bar{\mathbf{s}}^\alpha, \quad \mathbf{m}^\alpha = (\mathbf{F}^e)^{-T}\bar{\mathbf{s}}^\alpha,$$
$$\tau^\alpha = \frac{1}{\hat{\rho}_0}\left(\mathbf{S}(\mathbf{F}^p)^T\bar{\mathbf{m}}^\alpha\right) \cdot \mathbf{F}^e\bar{\mathbf{s}}^\alpha. \tag{12}$$

- *hardening law* is expressed either in terms of the dislocation densities, Cermelli and Bortoloni (2004):

$$\zeta^\alpha = \zeta^\alpha\left(\rho^\beta\right), \ \beta = 1, \ldots, N. \tag{13}$$

or by certain evolution equation, Teodosiu and Raphanel (1993)

$$\dot{\zeta}^\alpha = \sum_{\beta=1}^{N} h^{\alpha\beta}\left|\dot{\gamma}^\beta\right| \tag{14}$$

non-local evolution equation

- for the dislocation densities considered by Bortoloni and Cermelli (2004)

$$\dot{\rho}^{\alpha} = D\left|\nu^{\alpha}\right|\left(k\Delta\rho^{\alpha} - \frac{\partial\psi_T}{\partial\rho^{\alpha}}\right), \alpha = 1, \ldots, N \tag{15}$$

  where

$$\psi_T = \psi_T\left(\rho^{\beta}\right), \beta = 1, \ldots, N \tag{16}$$

  is a dislocation energy, D are k constant.

- boundary conditions

$$k\frac{\partial\rho^{\alpha}}{\partial n} = i^{\alpha}\left(\rho^{\beta}\right) \text{ pe } \partial B^{p}(t) \tag{17}$$

General problem

- $\mathbf{F}$, $\gamma^{\alpha}(i.e.\mathbf{F}^{p})$, ρ^{α}, for $\alpha = 1, \ldots, N$ defined on $B \times [0, T)$,
- equilibrium equation $div\mathbf{S} = 0$
- elastic type constitutive equation

$$\mathbf{S} = \mathbf{F}(\mathbf{F}^{p})^{-1}\left[\frac{1}{2}\lambda tr((\mathbf{F}^{p})^{-T}\mathbf{F}^{T}\mathbf{F}(\mathbf{F}^{p})^{-1} - \mathbf{I})\,\mathbf{I} + \right. \\ \left. +\mu\left((\mathbf{F}^{p})^{-T}\mathbf{F}^{T}\mathbf{F}(\mathbf{F}^{p})^{-1} - \mathbf{I}\right)\right](\mathbf{F}^{p})^{-T} \quad (18)$$

- Evolution equation for plastic distortion

$$\dot{\mathbf{F}}^{p} = \left[\sum_{\alpha=1}^{N}\left|\frac{\mathbf{S}(\mathbf{F}^{p})^{T}\bar{\mathbf{m}}^{\alpha}\cdot\mathbf{F}(\mathbf{F}^{p})^{-1}\bar{\mathbf{s}}^{\alpha}}{\zeta^{\alpha}(\rho^{\beta})}\right|^{n} H(\mathcal{F}^{\alpha})\,\mathrm{sgn}(\tau^{\alpha})\bar{\mathbf{s}}^{\alpha}\otimes\bar{\mathbf{m}}^{\alpha}\right]\mathbf{F}^{p}$$

- Evolution equations for the dislocation densities

$$\dot{\rho}^{\alpha} = D\left|\frac{\mathbf{S}(\mathbf{F}^{p})^{T}\bar{\mathbf{m}}^{\alpha}\cdot\mathbf{F}(\mathbf{F}^{p})^{-1}\bar{\mathbf{s}}^{\alpha}}{\zeta^{\alpha}(\rho^{\beta})}\right|^{n} H(\mathcal{F}^{\alpha})\left(k\Delta\rho^{\alpha} - \frac{\partial\psi_{T}}{\partial\rho^{\alpha}}\right), \ \alpha = 1, \ldots, N$$

- Boundary conditions

$$\mathbf{S}\mathbf{n} = \bar{\mathbf{t}}^0 \quad \text{on } \Gamma_1^0$$

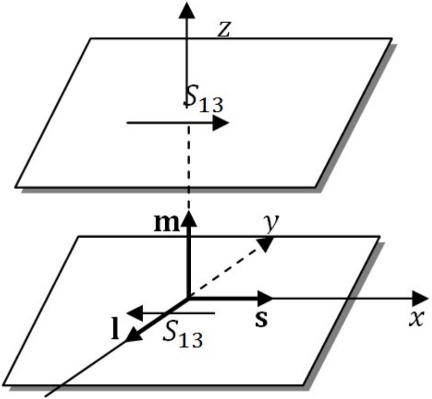
$$\mathbf{u} = \bar{\mathbf{u}} \text{ on } \Gamma_2^0 \tag{20}$$

$$k\frac{\partial \rho^\alpha}{\partial n} = i^\alpha(\rho^\beta) \quad \text{on } \partial B^p(t)$$

- Initial conditions

$$\gamma^\alpha(0) = 0 \quad \rho^\alpha(0) = \rho_0^\alpha \quad \alpha = 1, \ldots, N \quad \Longleftrightarrow \quad \mathbf{F}^p(0) = \mathbf{I} \tag{21}$$

Statement of the problem



domain occupied by the body is a layer B

$$B = \{(x, y, z) : 0 \leq z \leq L\} \quad (22)$$

- Only one slip system ($\bar{\mathbf{s}} \equiv \mathbf{i}, \quad \bar{\mathbf{m}} \equiv \mathbf{k}$,
- plastic distortion

$$\mathbf{F}^p = \mathbf{I} + \gamma \bar{\mathbf{s}} \otimes \bar{\mathbf{m}}, \quad \dot{\gamma} = \nu \tag{23}$$

- **Problem:** For a given homogeneous shear stress state,

$$\mathbf{S} = S_{13}(t)\,(\mathbf{i} \otimes \mathbf{k} + \mathbf{k} \otimes \mathbf{i}) \quad \text{with} \quad S_{13} : [t_0, T) \longrightarrow R_{\geq 0}$$

- find the unknowns

$$\mathbf{F} = \mathbf{F}\,(z,t)\,, \rho = \rho(z,t) \text{ si } \gamma = \gamma(z,t), \quad \text{defined on} \quad B \times [t_0, T)$$

- Elastic type constutive equation

$$S_{13}(t)\,(\mathbf{i} \otimes \mathbf{k} + \mathbf{k} \otimes \mathbf{i}) =$$
$$= \mathbf{F}(\mathbf{I} + \gamma \mathbf{s} \otimes \mathbf{m})^{-1} \left[\frac{1}{2} \lambda tr((\mathbf{I} + \gamma \mathbf{s} \otimes \mathbf{m})^{-T} \mathbf{F}^T \mathbf{F} (\mathbf{I} + \gamma \mathbf{s} \otimes \mathbf{m})^{-1} - \mathbf{I}) \right.$$
$$\left. + \mu \left((\mathbf{I} + \gamma \mathbf{s} \otimes \mathbf{m})^{-T} \mathbf{F}^T \mathbf{F} (\mathbf{I} + \gamma \mathbf{s} \otimes \mathbf{m})^{-1} - \mathbf{I} \right) \right] (\mathbf{I} + \gamma \mathbf{s} \otimes \mathbf{m})^{-T}$$

- Evolution equation for γ

$$\dot{\gamma} = \left| \frac{S_{13}(t)F_{11}}{\zeta(\rho)} \right|^n \text{sgn}\left(S_{13}(t)F_{11}\right) H(\mid \tau \mid -\zeta(\rho)) \tag{24}$$

- Evolution equation for the dislocation density

$$\dot{\rho} = D \left| \frac{S_{13}(t)F_{11}}{\zeta(\rho)} \right|^n \left(k\Delta\rho - \frac{\partial \psi_T}{\partial \rho} \right) H(\mid \tau \mid -\zeta(\rho)) \tag{25}$$

- Activation condition defined in terms of

$$\mid \tau \mid -\zeta(\rho) \equiv |S_{13}(t)F_{11}| - \zeta(\rho)\,. \tag{26}$$

- Initial condition

$$\gamma(z,0) = 0, \quad \rho(z,0) = \rho_0(z) \tag{27}$$

- boundary conditions

$$\frac{\partial \rho}{\partial z}(0,t) = \frac{\partial \rho}{\partial z}(L,t) = 0 \tag{28}$$

- liniar elastic response

$$\mathbf{S} = \mathbf{F}(\mathbf{F}^p)^{-1}\left(\lambda tr(\mathbf{E}^e)\mathbf{I} + 2\mu\mathbf{E}^e\right)(\mathbf{F}^p)^{-T}. \tag{29}$$

- Solve the problem for

$\mathbf{F}^p(t) = \mathbf{I}(i.e.\gamma(t) = 0), \rho(t) = \rho(0)$ on certain time interval $[0, t$

Theorem

Under the hypotheses det$\mathbf{F} > 0$ *and* $\mathbf{F}(0) = \mathbf{I}$, *the elastic solution has the form*

$$\mathbf{F} = \begin{pmatrix} F_{11} & F_{12} & F_{13} \\ 0 & F_{22} & 0 \\ F_{13} & F_{32} & F_{11} \end{pmatrix} \tag{30}$$

with

$$\begin{aligned} F_{22} &= \sqrt{F_{11}^2 + 3F_{13}^2} \\ F_{11} &= \sqrt{\frac{S_{13}}{2\mu F_{13}} + F_{13}^2}, \quad \textit{if} \quad F_{13} \neq 0, \end{aligned} \tag{31}$$

and $F_{13} = x$ *solution of the equation*

$$F_{13}^3 - \frac{(3\lambda + 2\mu)}{8(\lambda + \mu)} F_{13} + S_{13} \frac{(3\lambda + 2\mu)}{16\mu(\lambda + \mu)} = 0 \tag{32}$$

Theorem

Solution of the equation for $x = F_{13}$

$$x^3 + Ax + B = 0$$
$$with A = -\frac{(3\lambda + 2\mu)}{8(\lambda+\mu)}, \quad B = S_{13}\frac{(3\lambda+2\mu)}{16\mu(\lambda+\mu)} \equiv \frac{S_{13}}{2\mu}A \tag{33}$$

are given under the form

$$x_1 = -a - b$$
$$x_2 = \frac{a+b+\sqrt{-3(a-b)^2}}{2}, \quad x_3 = \frac{a+b-\sqrt{-3(a-b)^2}}{2} \tag{34}$$

where

$$b = \sqrt[3]{\frac{B-\sqrt{\Delta_t}}{2}}, \quad a = -\frac{A}{3\sqrt[3]{\frac{B+\sqrt{\Delta_t}}{2}}} \tag{35}$$

The solutions of the algebraic equation for F_{13} can be characterized through the signum of the expression

$$\Delta_t = B^2 + 4\left(\frac{A}{3}\right)^3 \equiv \Big\{\Big(\frac{S_{13}}{2\mu}\Big)^2 - \frac{2(3\lambda+2\mu)}{27(\lambda+\mu)}\Big\}\Big[\frac{3\lambda+2\mu}{16(\lambda+\mu)}\Big]^2 \quad (36)$$

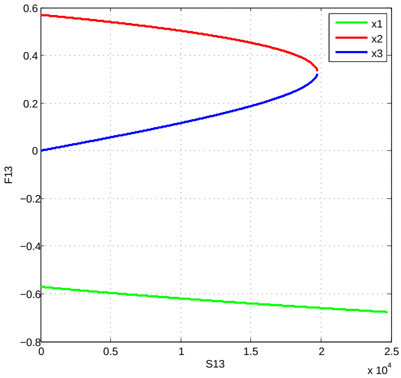
$$\Delta_t \leq 0 \quad \Longleftrightarrow S_{13} \leq \mu\sqrt{\frac{2}{27}\frac{3\lambda+2\mu}{\lambda+\mu}}$$

$$\Longleftrightarrow \quad \text{three real solutions exist, but either all are diffrent}$$

$$\Delta_t > 0 \quad \Longleftrightarrow S_{13} > \mu\sqrt{\frac{2}{27}\frac{3\lambda+2\mu}{\lambda+\mu}}$$

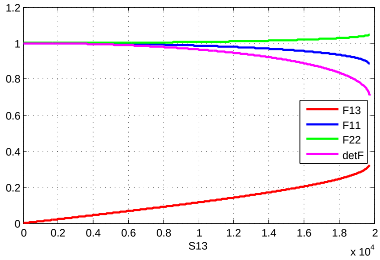
$$\Longleftrightarrow \text{only one real and two complex cojugated solutions e} \quad (37)$$

Elastic solution



The three possible elastic solutions, for shear component of deformation F_{13}

Elastic solution



The non-zero components of the elstic solution

Conclusions

To solve the general problem:

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1. First we solve the elastic solution in order to determine the critical stress state at which the activation criterion is reached, see the detail in 2.2.
2. Second we solve the sistem of equations, starting from the initial conditions that correspond to the critical stress state reached from the elastic solutions.

Conclusions

1. Graphics representations

- for the plastic shear γ,
- the dislocation densities ρ, and
  the non-vanishing components of the deformtion gradients,
  $F_{13}, F_{31}, F_{11} = F_{33}, F_{22}$

have been ploted in a spatial representation, as functions of the time t and the position z of the material points in the layer.

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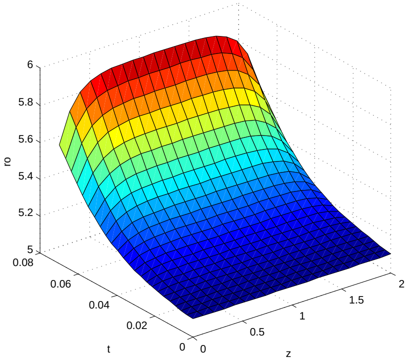
have been ploted in a spatial representation, as functions of the time t and the position z of the material points in the layer.

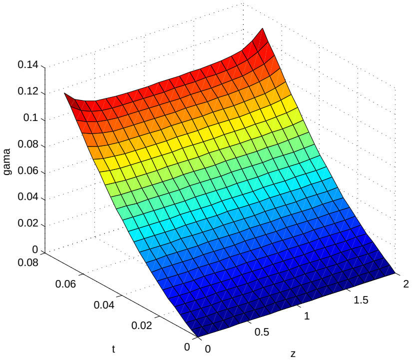
2. To develope a finite element description of the large deformation of a polycrystalline body
(following Teodosiu, Raphanel -1993, Simo, Hughes-2000)
for non-local models

Disclocation densities $\rho(t, z)$



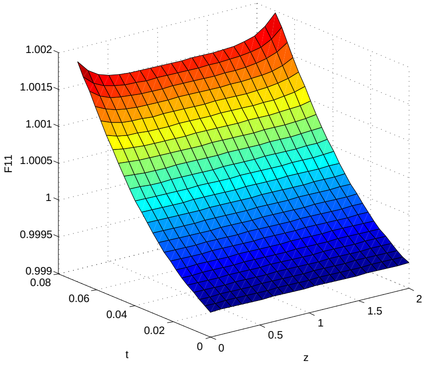
The evolution of non-homogeneous dislocation density

Plastic shear $\gamma(t, z))$



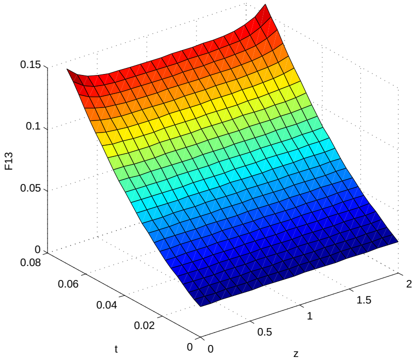
The evolution of non-homogeneous plastic shear

Non-zero components of the deformation gradient



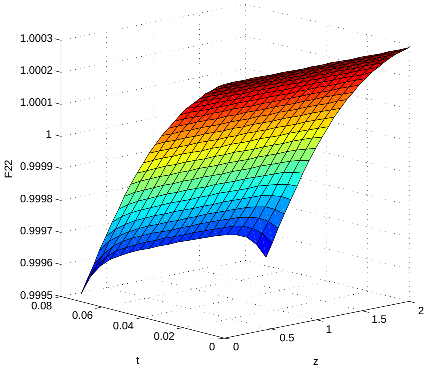
$F_{11}-$ component of the deformation gradient

Non-zero components of the deformation gradient $\mathbf{F}(t, z)$



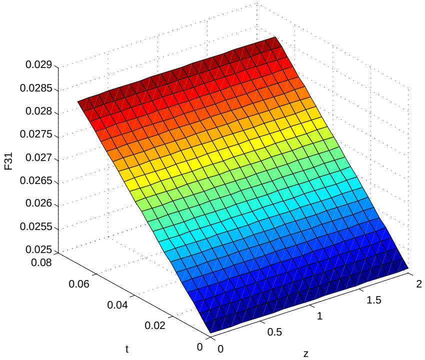
$F_{13}-$ component of the deformation gradient

Non-zero components of the deformation gradient $\mathbf{F}(t, z)$



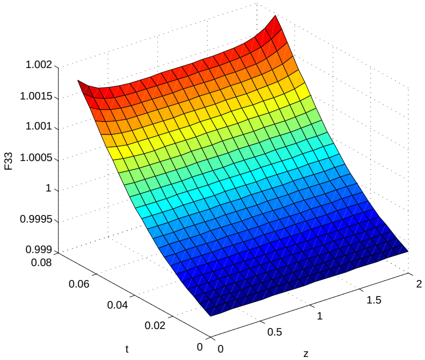
$F_{22}-$ component of the deformation gradient

Non-zero components of the deformation gradient $\mathbf{F}(t, z)$



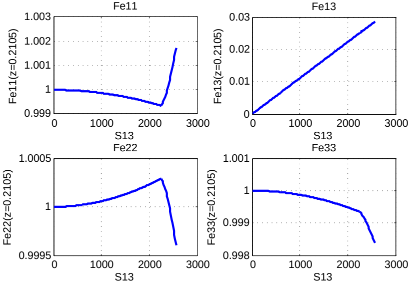
$F_{31}-$ component of the deformation gradient

Non-zero components of the deformation gradient $\mathbf{F}(t, z)$



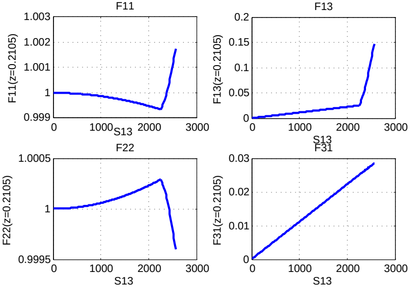
$F_{33}-$ component of the deformation gradient

Non-zero components of the deformation gradient for $z = 0.1205$



All non-vanishing components of the deformation gradient

Non-zero components of the elastic distorsion for $z = 0.1205$



All non-vanishing components of the deformation gradient

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