Continuum model of lattice defects in finite elasto-plasticity

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We describe the behaviour of the elasto-plastic material:

Ax. Based on the existence of configurations with torsion

$k-$ a fixed reference configuration of the body $\mathcal{B}$.

$\chi(\cdot, t)-$ the motion χ, at time t, for any $\mathbf{X} \in \mathcal{B}$

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- Evolution equations for $\mathcal{K}_t-$ have to be given.
- Energetic arguments: virtual power principle $\Longrightarrow$ macro and micro balance Eqns.

  energy imbalance $\Longrightarrow$ thermomechanics restrictions

We are not dealing with curved space but with curved geometry in flat space , de Wit (1981).

1. The nature of the geometry is determined by the linear connection $\mathbf{\Gamma}$, fixed by its coefficients
   the curvature tensor $\mathcal{R}$
   the Cartan torsion or torsion tensor $\mathbf{S}$;
2. metric tensor $\mathbf{C}$, to measure the distance;
3. non-metricity measure $\mathbf{Q}$, in terms of $\mathbf{\Gamma}$ and $\mathbf{C}$.

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- Geometry for which $\mathcal{R}, \mathbf{S}, \mathbf{Q}$ are non-vanishing is non-metric, non-Riemannian.
- If $\mathbf{Q} = 0$ the geometry is called metric.
- If $\mathcal{R} = 0$ the geometry is called flat.
- If $\mathbf{S} = 0$ the geometry is called symmetric.
- If $\mathbf{Q} = 0, \mathbf{S} = 0$ the geometry is called Riemannian.
- If $\mathcal{R} = 0, \mathbf{Q} = 0, \mathbf{S} = 0$ geomtry is called Euclidian.

- $\mathcal{B}$ is a continuous body of class C^2 if
  $\mathcal{B}$ is a n- differential manifold, dim $\mathcal{B}$= n,
  which is endowed with a structure by

(i) C a set of mappings, called configurations, i.e.
$\mathcal{C} := \{\phi : \mathcal{B} \longrightarrow \phi(\mathcal{B}) \mid \phi(\mathcal{B}) \subset \mathcal{E}, n -$ differential manifold
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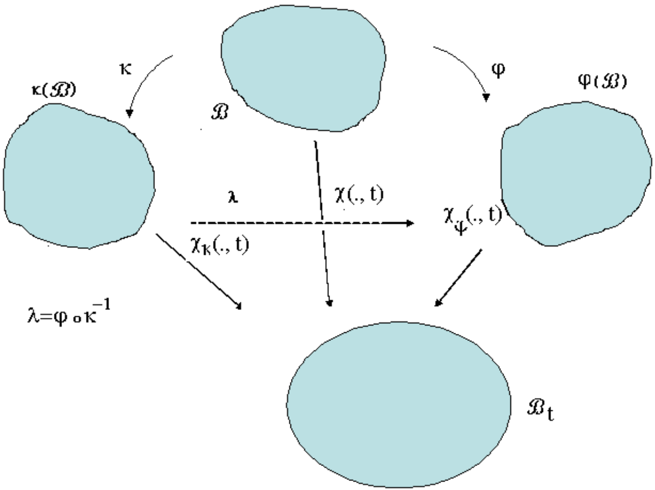
(ii) m a measure on $\mathcal{B}$, induced by the smooth density functions
$\rho_\kappa : \kappa(\mathcal{B}) \longrightarrow R_{>0}$, associated with any fixed configuration
$\kappa \in \mathcal{C}$, i.e.

$$m(\mathcal{P}) := m_\kappa(\kappa(\mathcal{P})) = \int_{\kappa(\mathcal{P})} \rho_\kappa(\mathbf{X}) dV_\kappa. \qquad (1)$$

$m(\mathcal{P})$ is called the mass of the part $\mathcal{P} \subset \mathcal{B}$.

Configurations and motion



Ax. (∃) Second order plastic deformation

$$\forall \quad \chi \text{motion of the body} \quad \mathcal{B} \quad \forall \mathbf{X}, \quad \forall t \quad \exists \quad (\mathbf{F}^p, \overset{(p)}{\mathbf{\Gamma}})$$

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$\mathbf{F}^p$ – an invertible second order tensor, i.e. $\mathbf{F}^p : \mathcal{T}_\mathbf{X} \to \mathcal{V}_\mathcal{K}$, called plastic distorsion,

- where $\mathcal{T}_\mathbf{X}$ – tangent space at $\mathbf{X}$, $\mathcal{V}_\mathcal{K}$ – a vector space,

$\overset{(p)}{\boldsymbol{\Gamma}}$ – third order field, $\boldsymbol{\Gamma}^p : \mathcal{T}_\mathbf{X} \longrightarrow Lin(\mathcal{T}_\mathbf{X}, \mathcal{T}_\mathbf{X})$, called plastic connection,

- with non-zeo torsion

$$(\mathbf{S}_k \mathbf{u})\mathbf{v} = (\overset{(p)}{\boldsymbol{\Gamma}}_k \mathbf{u})\mathbf{v} - (\overset{(p)}{\boldsymbol{\Gamma}}_k \mathbf{v})\mathbf{u} \quad \forall \mathbf{u}, \mathbf{v}. \tag{2}$$

$$\text{Calculus rule:} \nabla_\mathcal{K} \overline{\mathbf{F}} := (\nabla_k \overline{\mathbf{F}})(\mathbf{F}^p)^{-1}. \tag{3}$$

The composition rule of second order gradients

is reformulated for second order deformations

$$(\mathbf{F}, \mathbf{\Gamma}) := (\mathbf{F}^e, \overset{(e)}{\mathbf{\Gamma}}_{\mathcal{K}}) \circ (\mathbf{F}^p, \overset{(p)}{\mathbf{\Gamma}}_{k}), \quad \Longleftrightarrow$$

$$\mathbf{F} = \mathbf{F}^e\mathbf{F}^p, \quad \{\mathbf{F} = \nabla\chi\}$$

multiplicative decomposition (4)

$$\mathbf{\Gamma} = \mathbf{F}^p \overset{(e)}{\mathbf{\Gamma}}_{\mathcal{K}} [(\mathbf{F}^p)^{-1}, (\mathbf{F}^p)^{-1}] + \overset{(p)}{\mathbf{\Gamma}}_{k} \{= \quad \mathbf{F}^{-1}\nabla\mathbf{F}\}$$

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composition rule of the connections.

$$\text{Notation:}((\boldsymbol{\Gamma}[\mathbf{F}^p, \mathbf{F}^p])\mathbf{u})\mathbf{v} = (\boldsymbol{\Gamma}(\mathbf{F}^p\mathbf{u}))\mathbf{F}^p\mathbf{v}. \tag{5}$$

Plastic connection in $\mathcal{K}$

Ax. The plastic connection has non-metric property with respect to the appropriate plastic metric tensor $\mathbf{c}^p$,
(1) there exists $\mathbf{Q}^d_{\mathcal{K}}$ a third order tensor, such that

$$\mathbf{Q}^d_{\mathcal{K}}\tilde{\mathbf{u}} = \mathbf{c}^p \overset{(\mathrm{p})}{\mathbf{\Gamma}}_{\mathcal{K}} \tilde{\mathbf{u}} + (\overset{(\mathrm{p})}{\mathbf{\Gamma}}_{\mathcal{K}} \tilde{\mathbf{u}})^T \mathbf{c}^p - (\nabla_{\mathcal{K}} \mathbf{c}^p)\tilde{\mathbf{u}}, \tag{6}$$

$$\text{for} \qquad \mathbf{c}^p = (\mathbf{F}^p)^{-T}(\mathbf{F}^p)^{-1} \quad \text{the metric tensor in} \quad \mathcal{K}.$$

(2) $\mathbf{Q}^d_{\mathcal{K}}\tilde{\mathbf{u}} \in Sym, \quad \forall \quad \tilde{\mathbf{u}} \in \mathcal{V}.$
$\mathbf{Q}^d_{\mathcal{K}}$ is a measure of the non-metricity.

Theorem

The plastic connection with respect to $\mathcal{K}$ is a (1,2)- third order field, represented under the form

$$\mathbf{c}^p \overset{(p)}{\mathbf{\Gamma}}_{\mathcal{K}} = \mathbf{c}^p \overset{(p)}{\mathcal{A}}_{\mathcal{K}} + \frac{1}{2}\mathbf{Q}^d_{\mathcal{K}} + \mathbf{\Lambda}_{\mathcal{K}} \times \mathbf{I},$$

where $\overset{(p)}{\mathcal{A}}_{\mathcal{K}} := \mathbf{F}^p(\nabla_{\mathcal{K}}(\mathbf{F}^p)^{-1}),$ *Bilby's type connection*

$\mathbf{Q}^d_{\mathcal{K}}(\tilde{\mathbf{u}}) \in Sym, \quad \forall \quad \tilde{\mathbf{u}} \in \mathcal{V}_{\mathcal{K}},$ *non-metricity measure.*

The third order field $\mathbf{\Lambda}_{\mathcal{K}} \times \mathbf{I}$, *with* $\mathbf{\Lambda}_{\mathcal{K}}$ *a second order tensor field- the disclination tensor, is defined for any vectors* $\tilde{\mathbf{u}}, \tilde{\mathbf{v}}$, *by*

$$\begin{aligned} ((\mathbf{\Lambda}_{\mathcal{K}} \times \mathbf{I})\tilde{\mathbf{u}})\tilde{\mathbf{v}} &= (\mathbf{\Lambda}_{\mathcal{K}}\tilde{\mathbf{u}}) \times \tilde{\mathbf{v}}, \\ (\mathbf{\Lambda}_{\mathcal{K}} \times \mathbf{I})\tilde{\mathbf{u}} &= \mathbf{\Lambda}_{\mathcal{K}}\tilde{\mathbf{u}} \times \mathbf{I} \in Skew. \end{aligned} \tag{7}$$

Let us remark that Bilby's type connection $\overset{(\mathrm{p})}{\mathcal{A}}_{\mathcal{K}}$ is related to $\overset{(\mathrm{p})}{\mathcal{A}}_{k}$ by the plastic distorsion, as it follows

$$\overset{(\mathrm{p})}{\mathcal{A}}_{\mathcal{K}} = -\mathbf{F}^p \mathcal{A}^p_k[(\mathbf{F}^p)^{-1}, (\mathbf{F}^p)^{-1}] = 0, \qquad \text{with} \quad \overset{(\mathrm{p})}{\mathcal{A}}_{k} := (\mathbf{F}^p)^{-1} \nabla_k \mathbf{F}^p. \tag{8}$$

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2. The disclinations are characterized by a non-vanishing curature $\mathcal{R}$.
3. The extra-matter or vacancy are characterized by measures of non-metricity $\mathbf{Q}$.

Torsion of the plastic connection in $\mathcal{K}$

Let us introduce $\overset{(p)}{\bar{\mathbf{\Gamma}}}_{\mathcal{K}} := \mathbf{c}^p \overset{(p)}{\mathbf{\Gamma}}_{\mathcal{K}}$ a (0,3)-tensor, in $\mathcal{K}$

Definition

The Cartan torsion $\mathbf{S}_{\mathcal{K}}$, as a third order tensor, is given by

$$(\mathbf{S}^p_{\mathcal{K}}\tilde{\mathbf{u}})\tilde{\mathbf{v}} = (\overset{(p)}{\mathbf{\Gamma}}_{\mathcal{K}}\tilde{\mathbf{u}})\tilde{\mathbf{v}} - (\overset{(p)}{\mathbf{\Gamma}}_{\mathcal{K}}\tilde{\mathbf{v}})\tilde{\mathbf{u}} \tag{9}$$

The definition leads to the expression (written $\forall \quad \tilde{\mathbf{u}}, \tilde{\mathbf{v}}$)

$$((\bar{\mathbf{S}}^p_{\mathcal{K}})\tilde{\mathbf{u}})\tilde{\mathbf{v}} = (\mathbf{F}^p)^{-T}\mathrm{curl}_{\mathcal{K}}(\mathbf{F}^p)^{-1}(\mathbf{u}\times\mathbf{v}) + \frac{1}{2}((\mathbf{Q}^d_{\mathcal{K}}\tilde{\mathbf{u}})\tilde{\mathbf{v}} - (\mathbf{Q}^d_{\mathcal{K}}\tilde{\mathbf{v}})\tilde{\mathbf{u}}) +$$

$$+\mathbf{\Lambda}_{\mathcal{K}}\tilde{\mathbf{u}}\times\tilde{\mathbf{v}} - \mathbf{\Lambda}_{\mathcal{K}}\tilde{\mathbf{v}}\times\tilde{\mathbf{u}}, \quad \text{where} \quad \bar{\mathbf{S}}^p_{\mathcal{K}} = \mathbf{c}^p\mathbf{S}^p_{\mathcal{K}}.$$

Definition

The quasi-dislocation density (see Kröner, Anthony, de Wit) $\boldsymbol{\alpha}^{\mathbf{Q}}$ is a second order tensor

$$\text{with} \quad \boldsymbol{\alpha}^{\mathbf{Q}}(\tilde{\mathbf{u}} \times \tilde{\mathbf{v}}) := \frac{1}{2}\big((\mathbf{Q}^d_{\mathcal{K}}\tilde{\mathbf{u}})\tilde{\mathbf{v}} - (\mathbf{Q}^d_{\mathcal{K}}\tilde{\mathbf{u}})\tilde{\mathbf{v}}\big). \tag{10}$$

This is defined in analogy with the dislocation density $\boldsymbol{\alpha}$

$$\boldsymbol{\alpha}(\tilde{\mathbf{u}} \times \tilde{\mathbf{v}}) = (\mathbf{F}^p)^{-T}\mathrm{curl}_{\mathcal{K}}(\mathbf{F}^p)^{-1}(\mathbf{u} \times \mathbf{v}), \quad \forall \quad (\tilde{\mathbf{u}} \times \tilde{\mathbf{v}}). \tag{11}$$

Apart from the dislocation density which enters the definition of the Burgers vector, the quasi-dislocation density is a fictitious one.

Burgers vector

- in terms of plastic distorsion $\mathbf{F}^p$
- $\mathcal{A}_0$ surface with normal $\mathbf{N}$ bounded by C_0 a closed curve in k

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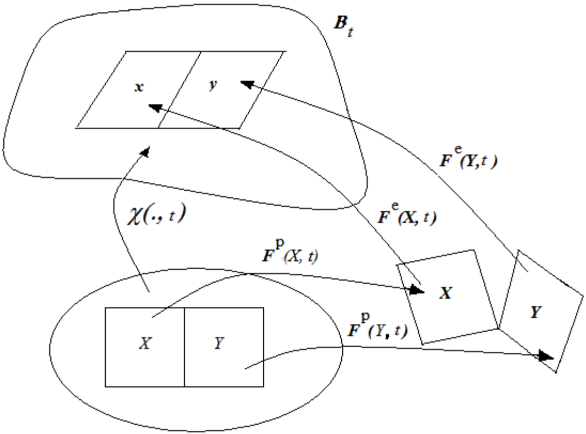
$$\begin{aligned} \mathbf{b}_{\mathcal{K}} &\equiv \{\int_{C_{\mathcal{K}}} d\mathbf{x}_{\mathcal{K}}\} = \int_{C_0} \mathbf{F}^p \, d\mathbf{X} = \\ &= \int_{\mathcal{A}_0} (\mathrm{curl}(\mathbf{F}^p))\mathbf{N} dA = \int_{\mathcal{A}_{\mathcal{K}}} \boldsymbol{\alpha}_{\mathcal{K}} \mathbf{n}_{\mathcal{K}} dA_{\mathcal{K}}, \end{aligned} \tag{12}$$

Burgers vector

- in terms of plastic distortion $\mathbf{F}^p$
- $\mathcal{A}_0$ surface with normal $\mathbf{N}$ bounded by C_0 a closed curve in k
- $$\begin{aligned} \mathbf{b}_{\mathcal{K}} &\equiv \{\int_{C_{\mathcal{K}}} d\mathbf{x}_{\mathcal{K}}\} = \int_{C_0} \mathbf{F}^p \, d\mathbf{X} = \\ &= \int_{\mathcal{A}_0} (\mathrm{curl}(\mathbf{F}^p))\mathbf{N} dA = \int_{\mathcal{A}_{\mathcal{K}}} \boldsymbol{\alpha}_{\mathcal{K}} \mathbf{n}_{\mathcal{K}} dA_{\mathcal{K}}, \end{aligned} \tag{12}$$
- $$\boldsymbol{\alpha}_{\mathcal{K}} \equiv \frac{1}{\det \mathbf{F}^p}(\mathrm{curl}(\mathbf{F}^p))(\mathbf{F}^p)^T \quad \text{Noll's disloc.} \tag{13}$$
  $$\mathbf{b}_{\mathcal{K}} \simeq \mathrm{curl}(\mathbf{F}^p)\mathbf{N} \; \mathrm{area}(\mathcal{A}_0)$$

Non-local config.: non-zero torsion and zero curvature



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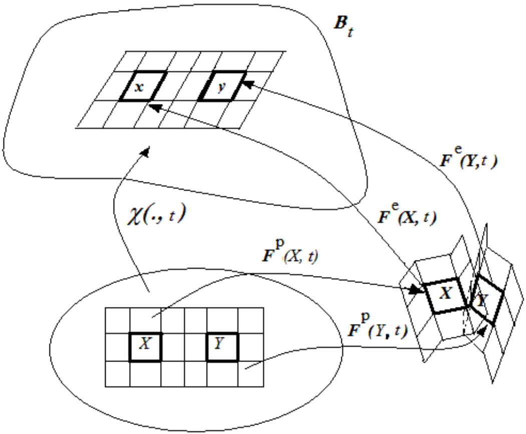

Definition

The second order torsion tensor $\mathcal{N}^p_{\mathcal{K}}$ is expressed by the dual representation, which relates Cartan torsion $\mathbf{S}_{\mathcal{K}}$ and $\mathcal{N}_{\mathcal{K}}$ by

$$(\mathbf{S}^p_{\mathcal{K}}\tilde{\mathbf{u}})\tilde{\mathbf{v}} = \mathcal{N}^p_{\mathcal{K}}(\tilde{\mathbf{u}} \times \tilde{\mathbf{v}}). \qquad (14)$$

Theorem

The second order torsion tensor $\bar{\mathcal{N}}^p_{\mathcal{K}}$ (where $\bar{\mathcal{N}}^p_{\mathcal{K}} = \mathbf{c}^p\mathcal{N}^p_{\mathcal{K}}$) is expressed by

$$\bar{\mathcal{N}}^p_{\mathcal{K}} = (\mathbf{F}^p)^{-T} curl_{\mathcal{K}}(\mathbf{F}^p)^{-1} + \boldsymbol{\alpha}^{\mathbf{Q}} + \left((tr\, \boldsymbol{\Lambda})\mathbf{I} - (\boldsymbol{\Lambda})^T\right). \qquad (15)$$

The following defect fields have been introduced

$\boldsymbol{\alpha}$	$:= (\mathbf{F}^p)^{-T}\mathrm{curl}_{\mathcal{K}}(\mathbf{F}^p)^{-1}$	dislocation density
$\boldsymbol{\alpha}^{\mathbf{Q}}$	associated with non-metricity	quasi-dislocation tensor
$\boldsymbol{\alpha}^{\Lambda}$	$:= \mathrm{tr}\, \boldsymbol{\Lambda}\mathbf{I} - (\boldsymbol{\Lambda})^T$	disclination density.

Disclination densities associated with non-metricity

in $\mathcal{K}$

Definition

$$curl\mathbf{Q}^d_{\mathcal{K}}\,(\tilde{\mathbf{u}}\times\tilde{\mathbf{v}}) = ((\nabla_{\mathcal{K}}\,\mathbf{Q}^d_{\mathcal{K}})\tilde{\mathbf{u}})\tilde{\mathbf{v}} - ((\nabla_{\mathcal{K}}\,\mathbf{Q}^d_{\mathcal{K}})\tilde{\mathbf{v}})\tilde{\mathbf{u}}. \quad (16)$$

Definition

The quasi- plastic strain $\mathbf{H}^{\mathbf{Q}}_{\mathcal{K}}$ is introduced through

$$\exists \quad \mathbf{H}^{\mathbf{Q}}_{\mathcal{K}} \in Sym \quad \text{such that} \quad \mathbf{Q}^d_{\mathcal{K}} = \nabla_{\mathcal{K}}\,\mathbf{H}^d_{\mathcal{K}} \quad \Longleftrightarrow \quad curl\mathbf{Q}^d_{\mathcal{K}} = 0 \quad (17)$$

- the expression for the quasi-dislocation in terms of the quasi-plastic strain $\mathbf{H}^d_{\mathcal{K}}$

The name of quasi-plastic strain for $\mathbf{H}^d_{\mathcal{K}}$ is justified through

$$\nabla_{\mathcal{K}} \quad (\mathbf{c}^p + \mathbf{H}^d_{\mathcal{K}})\tilde{\mathbf{u}} = \mathbf{c}^p \overset{(p)}{\mathbf{\Gamma}}_{\mathcal{K}} \tilde{\mathbf{u}} + (\overset{(p)}{\mathbf{\Gamma}}_{\mathcal{K}} \tilde{\mathbf{u}})^T \mathbf{c}^p \tag{19}$$

for $\qquad \mathbf{c}^p = (\mathbf{F}^p)^{-T}(\mathbf{F}^p)^{-1}$ the metric tensor in $\mathcal{K}$.

Remark. The covariant derivative of the metric tensor has to be corrected by the quasi-plastic tensor, which is only symmetric, apart from the the plastic metric tensor which is symmetric and positive definite tensor.

$$\mathbf{c}^p \overset{(p)}{\mathcal{A}}_{\mathcal{K}} \tilde{\mathbf{u}} + (\overset{(p)}{\mathcal{A}}_{\mathcal{K}} \tilde{\mathbf{u}})^T \mathbf{c}^p - (\nabla_{\mathcal{K}} \mathbf{c}^p)\tilde{\mathbf{u}}, \tag{20}$$

i.e. Bilby's connection $\mathcal{A}_{\mathcal{K}}$ has metric property relative to $\mathbf{c}^p$.

Definition

The Riemann curvature tensor $\mathcal{R}$ is defined, in a coordinate system, for any $\mathbf{u}, \mathbf{v}$, by

$$(\mathcal{R}\mathbf{u})\mathbf{v} = ((\nabla\mathbf{\Gamma})\mathbf{u})\mathbf{v} - ((\nabla\mathbf{\Gamma}\mathbf{v})\mathbf{u} + (\mathbf{\Gamma}\mathbf{u})\mathbf{\Gamma}\mathbf{v} - (\mathbf{\Gamma}\mathbf{v})\mathbf{\Gamma}\mathbf{u}. \qquad (21)$$

$\mathcal{R}^p_{\mathcal{K}}$ denotes the curvature tensor associated with the plastic connection relative to the configuration $\mathcal{K}$.
The non-metricity tensor $\mathbf{Q}^d_{\mathcal{K}}$ influences the Riemann curvature

$$\text{if} \quad curl\mathbf{Q}^d_{\mathcal{K}} \neq 0 \quad \text{then}$$

$$\frac{1}{2} curl\mathbf{Q}^d_{\mathcal{K}}(\tilde{\mathbf{u}} \times \tilde{\mathbf{v}}) =$$

$$= -\{\mathbf{c}^p(\mathcal{R}^p_{\mathcal{K}}\tilde{\mathbf{u}})\tilde{\mathbf{v}}\}^s - \left[\{(\mathbf{Q}_{\mathcal{K}}\tilde{\mathbf{u}}) \overset{(p)}{\bar{\mathbf{\Gamma}}}_{\mathcal{K}} \tilde{\mathbf{v}}\}^s - \{(\mathbf{Q}_{\mathcal{K}}\tilde{\mathbf{v}}) \overset{(p)}{\bar{\mathbf{\Gamma}}}_{\mathcal{K}} \tilde{\mathbf{u}}\}^s\right]$$

The disclination curvature tensor $\mathbf{r}^{\Lambda}_{\mathcal{K}}$

The expression of the curvature tensor that belongs to $\mathbf{\Lambda}$

$$(\bar{\mathcal{R}}^{\Lambda}_{\mathcal{K}}\tilde{\mathbf{u}})\tilde{\mathbf{v}} := \mathbf{c}^{p}(\mathcal{R}^{\Lambda}_{\mathcal{K}}\tilde{\mathbf{u}})\tilde{\mathbf{v}} = (\nabla_{\mathcal{K}}\ (\mathbf{\Lambda} \times \mathbf{I})\tilde{\mathbf{u}})\tilde{\mathbf{v}} - (\nabla_{\mathcal{K}}\ (\mathbf{\Lambda} \times \mathbf{I})\tilde{\mathbf{v}})\tilde{\mathbf{u}} +$$

$$+ (\mathbf{\Lambda}_{\mathcal{K}} \times \mathbf{I})\tilde{\mathbf{u}}(\mathbf{\Lambda}_{\mathcal{K}} \times \mathbf{I})\tilde{\mathbf{v}} - (\mathbf{\Lambda}_{\mathcal{K}} \times \mathbf{I})\tilde{\mathbf{v}}(\mathbf{\Lambda}_{\mathcal{K}} \times \mathbf{I})\tilde{\mathbf{u}},$$

$$\Longleftrightarrow \quad (\bar{\mathcal{R}}^{\Lambda}_{\mathcal{K}}\tilde{\mathbf{u}})\tilde{\mathbf{v}} = (curl_{\mathcal{K}}\ \mathbf{\Lambda}_{\mathcal{K}})(\tilde{\mathbf{u}} \times \tilde{\mathbf{v}}) + (Adj\ \mathbf{\Lambda}_{\mathcal{K}})^{T}(\tilde{\mathbf{u}} \times \tilde{\mathbf{v}}) \in Skew. \quad (22)$$

Adjoint of $\mathbf{\Lambda}$, denoted $Adj(\mathbf{\Lambda})$, is *defined*, as a second order tensor, by

$$\left(\mathbf{\Lambda}\tilde{\mathbf{u}}, \mathbf{\Lambda}\tilde{\mathbf{v}}, \tilde{\mathbf{w}}\right) := \left(\tilde{\mathbf{u}}, \tilde{\mathbf{v}}, (Adj\ \mathbf{\Lambda})\tilde{\mathbf{w}}\right), \quad \forall \quad \tilde{\mathbf{u}}, \tilde{\mathbf{v}}, \tilde{\mathbf{w}}. \qquad (23)$$

There exists a second order tensor $\mathbf{r}^{\Lambda}_{\mathcal{K}}$, such that

$$\mathbf{r}^{\Lambda}_{\mathcal{K}} = curl_{\mathcal{K}}\, \mathbf{\Lambda}_{\mathcal{K}} + (Adj\ \mathbf{\Lambda}_{\mathcal{K}})^{T}, \qquad \text{where} \quad \mathbf{r}^{\Lambda}_{\mathcal{K}}(\tilde{\mathbf{u}} \times \tilde{\mathbf{v}}) = (\bar{\mathcal{R}}^{\Lambda}_{\mathcal{K}} \tilde{\mathbf{u}})\tilde{\mathbf{v}}, \tag{24}$$

which is a measure of the Riemannian curvature.
In this case the lattice defect, the disclination $\mathbf{\Lambda}_{\mathcal{K}}$, leads to

- disclination density $\boldsymbol{\alpha}^{\Lambda} := (\text{tr}\ \mathbf{\Lambda})\mathbf{I} - (\mathbf{\Lambda})^{T}$,
- a non- zero curvature, which is characterized by the curvature second order tensor $\mathbf{r}^{\Lambda}_{\mathcal{K}}$.

Macro balance equations

1. balance of the linear momentum

$$\int_{\chi(\mathcal{P},t)} \rho \mathbf{a} dV = \int_{\chi(\mathcal{P},t)} \rho\, \mathbf{b} dV + \int_{\partial\chi(\mathcal{P},t)} \mathbf{t}\mathbf{n} dA. \qquad (25)$$

2. angular momentum

$$\int_{\chi(\mathcal{P},t)} \mathbf{r} \wedge \rho \mathbf{a} dV = \int_{\chi(\mathcal{P},t)} \mathbf{r} \wedge \rho\, \mathbf{b} dV + \int_{\partial\chi(\mathcal{P},t)} \mathbf{r} \wedge \mathbf{t}(\mathbf{n}) dA +$$

$$+ \int_{\chi(\mathcal{P},t)} \rho\, \mathbf{B}_m dV + \int_{\partial\chi(\mathcal{P},t)} \mathbf{M}(\mathbf{n}) dA,$$

Definition:

$\mathbf{r} \wedge \mathbf{a} \in Skew, \quad (\mathbf{r} \wedge \mathbf{a})\mathbf{w} = (\mathbf{r} \times \mathbf{a}) \times \mathbf{w}, \quad \forall \quad \mathbf{r}, \mathbf{a}, \mathbf{w} \in \mathcal{V}.$

- local balance law for linear momentum

$$\text{div}\ \mathbf{T} + \rho\mathbf{b} = \rho\ \mathbf{a}, \tag{27}$$

  - Cauchy stress $\mathbf{T}$, generally non-symmetric,
  - third order tensor $\boldsymbol{\mu}-$ macro momentum $\mathbf{M}(\mathbf{n}) = \boldsymbol{\mu}\mathbf{n}$.

$$\begin{aligned}\int_{\chi(\mathcal{P},t)} \mathbf{r}\ \wedge\ \text{div}\mathbf{T}dV = \int_{\partial\chi(\mathcal{P},t)} \mathbf{r}\ \wedge \mathbf{t}\ (\mathbf{n})dA + \\ + \int_{\chi(\mathcal{P},t)} (\text{div}\ \boldsymbol{\mu} + \rho\ \mathbf{B}_m)dV.\end{aligned} \tag{28}$$

- local balance law for angular momentum

$$\int_{\chi(\mathcal{P},t)} (-2\mathbf{T}^a + \text{div}\ \boldsymbol{\mu} + \rho\ \mathbf{B}_m)dV = 0. \tag{29}$$

local balance equations for the linear and angular momentum

$$\begin{aligned} &\mathrm{div}\ \mathbf{T} + \rho\mathbf{b} = \rho\mathbf{a}, \\ &-2\mathbf{T}^a = \mathrm{div}\ \boldsymbol{\mu} + \rho\mathbf{B}_m, \quad -2\mathbf{T}^a = \mathbf{T}^*. \end{aligned} \tag{30}$$

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the balance equations

$$\begin{aligned} &\text{div } \left(\mathbf{T}^s - \frac{1}{2}\{\text{div } \boldsymbol{\mu}\}^a\right) + \rho \mathbf{b} = \rho \mathbf{a} \\ &\text{with the compatibility condition } \{\text{div } \boldsymbol{\mu}\}^s + \rho \mathbf{B}_m = 0, \end{aligned} \tag{31}$$

when the body momentum is symmetric, i.e. $\mathbf{B}_m \in Sym$.

Balance equations for micro forces

1. The local micro balance equation involving plastic micro forces

$$\boldsymbol{\Upsilon}^p_{\mathcal{K}} = \text{div}\,(\boldsymbol{\mu}^p_{\mathcal{K}} - \boldsymbol{\mu}_{\mathcal{K}}) + \tilde{\rho} B^p_m, \quad \text{in } \mathcal{K}(\mathcal{P}, t), \tag{32}$$

2. The micro balance equation for micro forces associated with the disclination

$$\begin{aligned} \boldsymbol{\Upsilon}^\lambda - \text{div}_{\mathcal{K}}\, \boldsymbol{\mu}^\lambda &= 0, \\ \boldsymbol{\Upsilon}^Q_{\mathcal{K}} - div\, \boldsymbol{\mu}^Q_{\mathcal{K}} &= 0 \end{aligned} \tag{33}$$

Here $\tilde{\rho}\mathbf{B}^\lambda$ is mass density of the couple body force, $\boldsymbol{\Upsilon}^\lambda$ are micro stress and $\boldsymbol{\mu}^\lambda$ are micro momentum associated with the disclinations.

Free energy density

Ax. If disclinations and extra-matter defects are considered, then there exists a free density energy function ψ, represented in $\mathcal{K}$ by

$$\psi = \psi_{\mathcal{K}}(\mathbf{C}^e, \overset{(e)}{\mathcal{A}}_{\mathcal{K}}, (\mathbf{F}^p)^{-1}, \overset{(p)}{\mathcal{A}}_{\mathcal{K}}, \mathbf{H}^Q, \nabla_{\mathcal{K}}\mathbf{H}^Q, \mathbf{\Lambda}_{\mathcal{K}}, \nabla_{\mathcal{K}}\mathbf{\Lambda}_{\mathcal{K}}) \tag{34}$$

as a function dependent on

- the second order elastic deformation $(\mathbf{C}^e, \overset{(e)}{\mathcal{A}}_{\mathcal{K}})$
- the plastic measure of deformation $((\mathbf{F}^p)^{-1}, \overset{(p)}{\mathcal{A}}_{\mathcal{K}})$
- the quasi-plastic strain $\mathbf{H}^Q \in Sym$ and its gradient
- the disclination variable $\mathbf{\Lambda}_{\mathcal{K}}$ and its gradient.

Imbalanced free energy

Ax. The virtual internal power in $\mathcal{K}$

$$virt(\mathcal{P}_{int})_{\mathcal{K}} = \frac{1}{\rho}(\mathbf{T}+\mathbf{T}^*)\cdot\widetilde{\mathbf{L}}^e + \frac{1}{\rho_{\mathcal{K}}}\boldsymbol{\mu}_{\mathcal{K}}\cdot virt\mathcal{L}_{\mathbf{L}^p}[\overset{(e)}{\mathcal{A}_{\mathcal{K}}}]+$$

$$+\frac{1}{\rho_{\mathcal{K}}}\boldsymbol{\Upsilon}^p_{\mathcal{K}}\cdot\widetilde{\mathbf{L}}^p + \frac{1}{\rho_{\mathcal{K}}}\boldsymbol{\mu}^p_{\mathcal{K}}\cdot\nabla_{\mathcal{K}}\widetilde{\mathbf{L}}^p + \frac{1}{\rho_{\mathcal{K}}}\boldsymbol{\Upsilon}^\lambda_{\mathcal{K}}\cdot(\delta\boldsymbol{\Lambda})+$$

$$+\frac{1}{\rho_{\mathcal{K}}}\boldsymbol{\mu}^\Lambda_{\mathcal{K}}\cdot\nabla_{\mathcal{K}}\delta\boldsymbol{\Lambda} + \frac{1}{\rho_{\mathcal{K}}}\boldsymbol{\mu}^Q_{\mathcal{K}}\cdot\nabla_{\mathcal{K}}\delta\mathbf{H}^Q + \frac{1}{\rho_{\mathcal{K}}}\boldsymbol{\Upsilon}^Q_{\mathcal{K}}\cdot\delta\mathbf{H}^Q.$$

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$$+\frac{1}{\rho_{\mathcal{K}}}\boldsymbol{\mu}^{\Lambda}_{\mathcal{K}}\cdot\nabla_{\mathcal{K}}\delta\boldsymbol{\Lambda} + \frac{1}{\rho_{\mathcal{K}}}\boldsymbol{\mu}^{Q}_{\mathcal{K}}\cdot\nabla_{\mathcal{K}}\delta\mathbf{H}^{Q} + \frac{1}{\rho_{\mathcal{K}}}\boldsymbol{\Upsilon}^{Q}_{\mathcal{K}}\cdot\delta\mathbf{H}^{Q}.$$

Ax. The elasto-plastic behavior of the material is restricted to satisfy in $\mathcal{K}$ the imbalanced free energy condition

$$-\dot{\psi}_{\mathcal{K}} + (\mathcal{P}_{int})_{\mathcal{K}} \geq 0 \quad \text{for any virtual (isothermic) processes.} \quad (35)$$

Discilination only is associated with dislocation mechanism

- The free energy in the reference configuration when the disclination density $\widetilde{\mathbf{\Lambda}} \equiv \Lambda_{\mathcal{K}}$ is involved

$$\psi = \psi(\mathbf{C}, \mathbf{\Gamma}, \mathbf{F}^p, \overset{(\mathrm{p})}{\mathcal{A}}, \mathbf{\Lambda}, \nabla\mathbf{\Lambda}) \equiv \psi_{\mathcal{K}}(\mathbf{C}^e, \overset{(\mathrm{e})}{\mathcal{A}}_{\mathcal{K}}, (\mathbf{F}^p)^{-1}, \overset{(\mathrm{p})}{\mathcal{A}}_{\mathcal{K}}, \widetilde{\mathbf{\Lambda}}, \nabla_{\mathcal{K}}\widetilde{\mathbf{\Lambda}}).$$

- Restrictions imposed by the imbalanced free energy to the elastic type constitutive functions are

$$\frac{1}{\rho}\{\mathbf{T}\}^s = 2\mathbf{F}(\partial_{\mathbf{C}}\psi)\mathbf{F}^T, \quad \frac{1}{\rho_0}\boldsymbol{\mu}_0 = \partial_{\mathbf{\Gamma}}\psi.$$

Viscoplastic type constitutive equations for micro forces

contain:

1. a dissipative part
2. a non-dissipative part, which is derived from the free energy, the so-called energetic micro forces,

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2. a non-dissipative part, which is derived from the free energy, the so-called energetic micro forces,
   The micro forces assocciated with plastic mechanism being represented through

$$\frac{1}{\rho_0}(\boldsymbol{\Sigma}_0 - \boldsymbol{\Sigma}_0{}^p) + (\mathbf{F}^p)^T \partial_{\mathbf{F}^p}\psi + \overset{(p)}{\mathcal{A}} \odot \left(\frac{1}{\rho_0}(\boldsymbol{\mu}_0 - \boldsymbol{\mu}_0{}^p) + \partial_{\overset{(p)}{\mathcal{A}}}\psi\right) -$$

$$-\left(\frac{1}{\rho_0}(\boldsymbol{\mu}_0 - \boldsymbol{\mu}_0{}^p) + \partial_{\overset{(p)}{\mathcal{A}}}\psi\right) {}_r\odot \overset{(p)}{\mathcal{A}} = \xi_1 \, \mathbf{l}^p, \tag{36}$$

$$\frac{1}{\rho_0}(\boldsymbol{\mu}_0 - \boldsymbol{\mu}_0{}^p) - \partial_{\overset{(p)}{\mathcal{A}}}\psi = \xi_2 \, \nabla \mathbf{l}^p,$$

Viscoplastic type constitutive equations for micro forces

The micro forces assocciated with disclinations are characterized by

$$\frac{1}{\rho_0}\boldsymbol{\mu_0}^{\lambda} - \partial_{\nabla\boldsymbol{\Lambda}}\,\psi = \xi_4\,\nabla\dot{\boldsymbol{\Lambda}},$$

$$\Big(\frac{1}{\rho_0}\boldsymbol{\Sigma_0}^{\lambda} - \partial_{\boldsymbol{\Lambda}}\,\psi\Big) + \Big(\overset{(\mathrm{p})}{\mathcal{A}}\odot\frac{1}{\rho_0}\boldsymbol{\mu}_0^{\lambda}\Big) - \qquad (37)$$

$$-\Big(\frac{1}{\rho_0}\boldsymbol{\mu}_0^{\lambda}\;{}_r\!\odot\overset{(\mathrm{p})}{\mathcal{A}}\Big) - \frac{1}{\rho_0}\boldsymbol{\mu}_0^{\lambda}\big(\mathrm{tr}_{(2)}(\overset{(\mathrm{p})}{\mathcal{A}})\big) = \xi_3\,\dot{\boldsymbol{\Lambda}}.$$

special case

$$\frac{1}{\rho_0}\boldsymbol{\mu_0}^{\lambda} - \partial_{\nabla\boldsymbol{\Lambda}}\,\psi = 0$$

Shear plastic distortion as a source of disclination

1. If the plastic distortion $\mathbf{F}^p$ corresponds to a simple shear in the slip system $\mathbf{s}, \mathbf{m}$, with ξ be the direction of the dislocation line

$$\mathbf{F}^p = \mathbf{I} + \gamma \mathbf{s} \otimes \mathbf{m}, \quad \mathbf{s} \perp \mathbf{m},$$
$$\nabla \mathbf{F}^p = \mathbf{s} \otimes \mathbf{m} \otimes \nabla \gamma, \quad J^p := det(\mathbf{F}^p) = 1, \tag{38}$$

2. The dislocation density tensor can be represented in terms of edge and screw dislocations

$$\boldsymbol{\alpha} := \rho_{\perp} \mathbf{b} \otimes \boldsymbol{\xi} + \rho_{\odot} \mathbf{b} \otimes \mathbf{b}, \quad \mathbf{b} - \text{Burgers vector}$$
$$\text{where} \quad \mathbf{b} := \mathbf{e}_1 = \mathbf{s}, \quad \boldsymbol{\xi} = \mathbf{e}_2, \quad \mathbf{e}_3 := \mathbf{m}. \tag{39}$$

3. the Bilby's type plastic connection is considered

$$\mathcal{A}^p := (\mathbf{F}^p)^{-1} \nabla \mathbf{F}^p = \mathbf{e}_1 \otimes \mathbf{e}_3 \otimes \nabla \gamma, \quad \nabla \gamma = \frac{\partial \gamma}{\partial x^1} \mathbf{e}_1 + \frac{\partial \gamma}{\partial x^2} \mathbf{e}_2.$$

- $\boldsymbol{\Lambda}$ represented in the specific form

$$\boldsymbol{\Lambda} := \eta \boldsymbol{\omega} \otimes \boldsymbol{\zeta},$$

$\boldsymbol{\omega}-$ Frank vector
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$\boldsymbol{\omega}-$ Frank vector
$\boldsymbol{\zeta}-$ the tangent vector line for the disclination,

- to be solution of the micro balance equation for the disclination in k

$$J^p\, \boldsymbol{\Upsilon}^\lambda = \mathrm{div}(J^p\, \boldsymbol{\mu}^\lambda (\mathbf{F}^p)^{-T}) + \rho_0 \mathbf{B}^\lambda, \qquad (40)$$

$J^p\, \boldsymbol{\mu}^\lambda (\mathbf{F}^p)^{-T}$ defines a micro momentum associated with the disclination in k,

- with the appropriate boundary conditions on $\partial\mathcal{K}(\mathcal{P}, t)$,
- and satisfying the appropriate evolution equation.

Theorem

1. Under the hypothesis that $(\mathbf{s}\cdot\omega)(\zeta\cdot\mathbf{m})$ *is not vanishing, then the evolution equation for the density of disclination is given by*

$$\xi_3\dot{\eta} + \kappa_2\beta_2^2\Delta\eta = 0, \tag{41}$$

while the compatibility condition expressed through the orthogonality condition

$$\nabla\gamma\cdot\nabla\eta = 0. \tag{42}$$

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2. The result remains still valuable if either $(\mathbf{s}\cdot\boldsymbol{\omega}) = 0$ *or* $(\zeta\cdot\mathbf{m}) = 0$.
3. If both $(\mathbf{s}\cdot\boldsymbol{\omega}) = 0, \quad (\zeta\cdot\mathbf{m}) = 0,$ *then (41) holds, without any restriction relative to plastic shear* γ.

Conclusions

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Conclusions

- We developed a general mathematical framework, able to cover a large range of second order plasticity,
- based on anholonomic configuration,
- taking into account the presence of the inhomogeneities, which describe lattice defects
- such as continuously distributed dislocation, disclinations, and extra-matter.
- Dislocations can be represented by the curl of the plastic distorsion, disclinations are characterized by a second order tensor viewed as a measure of non-zero curvature and being different from the Riemannian one, while the extra-matter can be related to quasi-plastic strain, as a measure of non-metricity.

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